PART : MATHEMATICS

SECTION – 1

This section contains **19 multiple choice questions.** Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which **Only One** is correct.

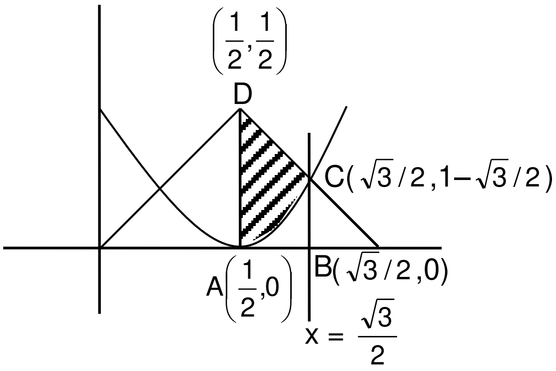
1. If f(x) = $\begin{cases} x & 0 < x < \frac{1}{2} \\ \frac{1}{2} & x = \frac{1}{2} \\ 1 - x & \frac{1}{2} < x < 1 \end{cases}$

g(x) = $\left(x - \frac{1}{2}\right)^2$ then find the area bounded by f(x) and g(x) from x = $\frac{1}{2}$ to x = $\frac{\sqrt{3}}{2}$.

(1) $\frac{\sqrt{3}}{4} - \frac{1}{3}$ (2) $\frac{\sqrt{3}}{4} + \frac{1}{3}$ (3) $2\sqrt{3}$ (4) $3\sqrt{3}$

Ans. **(1)**

Sol.



Required area = Area of trapezium ABCD – $\int_{1/2}^{\sqrt{3}/2} \left(x - \frac{1}{2}\right)^2 dx$

$$= \frac{1}{2}\left(\frac{\sqrt{3}-1}{2}\right)\left(\frac{1}{2}+1-\frac{\sqrt{3}}{2}\right) - \frac{1}{3}\left(\left(x-\frac{1}{2}\right)^3\right)_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}}$$

$$= \frac{\sqrt{3}}{4} - \frac{1}{3}$$

2. z is a complex number such that |Re(z)| + |Im (z)| = 4 then |z| can't be

(1) $\sqrt{7}$ (2) $\sqrt{10}$ (3) $\sqrt{\frac{17}{2}}$ (4) $\sqrt{8}$

Ans. **(1)**

Sol. z = x + iy

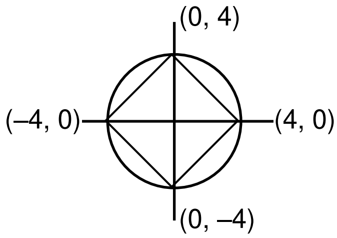
|x| + |y| = 4

Minimum value of |z| = $2\sqrt{2}$

Maximum value of |z| = 4

$|z| \in \left[\sqrt{8}, \sqrt{16}\right]$

So |z| can't be $\sqrt{7}$



Sol.

3. If $f(x) = \begin{vmatrix} x+a & x+2 & x+1 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}$ and $a - 2b + c = 1$ then

(1) $f(50) = 1$ (2) $f(-50) = -1$

(3) $f(50) = 501$ (4) $f(50) = -501$

Ans. **(1)**

Sol. Apply $R_1 = R_1 + R_3 - 2R_2$ प्रयोग करने पर

$$\Rightarrow f(x) = \begin{vmatrix} 1 & 0 & 0 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix} \Rightarrow f(x) = 1 \qquad \Rightarrow f(50) = 1$$

4. Let a_n is a positive term of a GP and $\sum_{n=1}^{100} a_{2n+1} = 200$, $\sum_{n=1}^{100} a_{2n} = 100$ find $\sum_{n=1}^{200} a_n$

(1) 300 (2) 150 (3) 175 (4) 225

Ans. **(2)**

Sol. Let GP is a, ar, ar^2 ……..

$$\sum_{n=1}^{100} a_{2n+1} = a_3 + a_5 + \ldots\ldots a_{201} = 200 \Rightarrow \frac{ar^2(r^{200}-1)}{r^2-1} = 200 \qquad \ldots.(1)$$

$$\sum_{n=1}^{100} a_{2n} = a_2 + a_4 + \ldots\ldots. a_{200} = 100 = \frac{ar(r^{200}-1)}{r^2-1} = 100 \qquad \ldots.(2)$$

Form (1) and (2) r = 2

add both

$\Rightarrow a_2 + a_3 + \ldots\ldots\ldots.. a_{200} + a_{201} = 300 \quad \Rightarrow \quad r(a_1 + \ldots\ldots\ldots.. a_{200}) = 300$

$$\sum_{n=1}^{200} a_n = \frac{300}{r} = 150$$

Sol.

5. If $\frac{}{dx} = \frac{xy}{x^2+y^2}$, y(1) = 1 and y(x) = e then x = ?

(1) $\frac{\sqrt{3}}{2}e$ (2) $\sqrt{3}\,e$ (3) $\sqrt{2}\,e$ (4) $\frac{e}{\sqrt{2}}$

Ans. (2)

Sol. Put y = vx

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = \frac{vx^2}{x^2+v^2x^2}$$

$$\Rightarrow \quad \frac{1+v^2}{v^3}dv = -\frac{1}{x}dx$$

$$\Rightarrow \quad \int\left(\frac{1}{v^3}+\frac{1}{v}\right)dv = \int\frac{-1}{x}dx$$

$$\Rightarrow \quad \frac{-1}{2}\,\frac{1}{v^2} + \ell nv = -\ell nx + c$$

$$\Rightarrow \quad -\frac{x^2}{2y^2} = -\ell ny + c$$

When x = 1, y = 1 then

$$-\frac{1}{2} = c$$

$\Rightarrow$ $x^2 = y^2(1 + 2\ell ny)$

$\Rightarrow$ $x^2 = e^2(3)$

$\Rightarrow$ $x = \pm\sqrt{3}\,e$

So इसलिये $x = \sqrt{3}e$

6. Let probability distribution is

x_i :	1	2	3	4	5
P_i :	k^2	2k	k	2k	$5k^2$

then value of p(x > 2) is

(1) $\frac{7}{12}$ (2) $\frac{1}{36}$ (3) $\frac{1}{6}$ (4) $\frac{23}{36}$

Ans. **(4)**

Sol. $\sum p_i = 1 \Rightarrow 6k^2 + 5k = 1$

$6k^2 + 5k - 1 = 0$

$6k^2 + 6k - k - 1 = 0$

$(6k - 1)(k + 1) = 0 \Rightarrow k = -1$ (rejected) ; $k = \frac{1}{6}$

$P(x > 2) = k + 2k + 5k^2$

$= \frac{1}{6}+\frac{2}{6}+\frac{5}{36} = \frac{6+12+5}{36} = \frac{23}{36}$

7. $\int \frac{d\theta}{\cos^2\theta(\sec 2\theta + \tan 2\theta)} = \lambda\tan\theta + 2\log f(x) + c$ then ordered pair $(\lambda, f(x))$ is

$\int \frac{d\theta}{\cos^2\theta(\sec 2\theta + \tan 2\theta)} = \lambda\tan\theta + 2\log f(x) + c$ $(\lambda, f(x))$ है–

(1) $(1, 1 + \tan\theta)$ (2) $(1, 1 - \tan\theta)$ (3) $(-1, 1 + \tan\theta)$ (4) $(-1, 1 - \tan\theta)$

Ans. (3)

Sol. $\int \frac{\sec^2\theta}{\frac{1+\tan^2\theta}{1-\tan^2\theta}+\frac{2\tan\theta}{1-\tan^2\theta}}\, d\theta$

$= \int \frac{\sec^2\theta(1-\tan^2\theta)}{(1+\tan\theta)^2} d\theta$

$= \int \frac{\sec^2\theta(1-\tan\theta)}{1+\tan\theta} d\theta$

$\tan\theta = t \Rightarrow \sec^2\theta\, d\theta = dt$

$= \int\left(\frac{1-t}{1+t}\right)dt = \int\left(-1+\frac{2}{1+t}\right)dt$

$= -t + 2\log(1+t) + C$

$= -\tan\theta + 2\log(1 + \tan\theta) + C$

$\Rightarrow \quad \lambda = -1$ and $f(x) = 1 + \tan\theta$

8. If $p \to (p \wedge \sim q)$ is false. Truth value of p & q will be

यदि $p \to (p \wedge \sim q)$ असत्य है, तब p & q

(1) TT (2) TF (3) F T (4) F F

Ans. (1)

Sol.

p	q	~q	p∧~q	p→(p∧~q)
T	T	F	F	F
T	F	T	T	T
F	T	F	F	T
F	F	T	F	T

9. If $\lim\limits_{x\to 0} x\left[\dfrac{4}{x}\right]$ = A then the value of x at which f(x) = $[x^2]\sin\pi x$ is discontinuous (where [.] denotes greatest integer function)

(1) $\sqrt{A+1}$ (2) $\sqrt{A+21}$ (3) $\sqrt{A}$ (4) $\sqrt{A+5}$

Ans. (1)

Sol. $\lim\limits_{x\to 0} x\left(\dfrac{4}{x}-\left\{\dfrac{4}{x}\right\}\right) = A \Rightarrow \lim\limits_{x\to 0} 4 - x\left\{\dfrac{4}{x}\right\}$ $\Rightarrow 4 - 0 = A$

check when जाँचने पर

(A) x = $\sqrt{A+1} \Rightarrow x = \sqrt{5} \Rightarrow$ discontinuous

(B) x = $\sqrt{A+21} \Rightarrow x = 5 \Rightarrow$ continuous

(C) x = $\sqrt{A} \Rightarrow x = 2 \Rightarrow$ continuous

(D) x = $\sqrt{A+5} \Rightarrow x = 3 \Rightarrow$ continuous

10. Let one end of focal chord of parabola $y^2 = 8x$ is $\left(\dfrac{1}{2}, -2\right)$, then equation of tangent at other end of this focal chord is

(1) x + 2y + 8 = 0 (2) x + 2y = 8 (3) x – 2y = 8 (4) x – 2y + 8 = 0

Ans. **(4)**

Sol. Let $\left(\dfrac{1}{2}, -2\right)$ is $(2t^2, 4t) \Rightarrow t = \dfrac{-1}{2}$

Parameter of other end of focal chord is 2

$\Rightarrow$ point is (8, 8)

$\Rightarrow$ equation of tangent is 8y – 4(x+8) = 0

$\Rightarrow$ 2y – x = 8

11. Let x + 6y = 8 is tangent to standard ellipse where minor axis is $\frac{4}{\sqrt{3}}$, then eccentricity of ellipse is

(1) $\sqrt{\frac{5}{6}}$ (2) $\sqrt{\frac{11}{12}}$ (3) $\frac{1}{3}\sqrt{\frac{11}{3}}$ (4) $\frac{1}{4}\sqrt{\frac{11}{12}}$

Ans. (2)

Sol. $2b = \frac{4}{\sqrt{3}} \Rightarrow b = \frac{2}{\sqrt{3}}$

Equation of tangent $\equiv y = mx \pm \sqrt{a^2m^2 + b^2}$

comparing with $\equiv y = \frac{-x}{6} + \frac{4}{3}$

$\equiv y = \frac{-x}{6} + \frac{4}{3}$

$m = \frac{-1}{6}$ and तथा $a^2m^2 + b^2 = \frac{16}{9}$

$\Rightarrow \frac{a^2}{36} + \frac{4}{3} = \frac{16}{9}$

$\Rightarrow \frac{a^2}{36} = \frac{16}{9} - \frac{4}{3} = \frac{4}{9}$

$\Rightarrow a^2 = 16$

$e = \sqrt{1 - \frac{b^2}{a^2}}$

$e = \sqrt{1 - \frac{4}{3 \times 16}} = \sqrt{\frac{11}{12}}$

12. if f(x) and g(x) are continuous functions, fog is identity function, $g'(b) = 5$ and $g(b) = a$ then $f'(a)$ is

(1) $\frac{2}{5}$ (2) $\frac{1}{5}$ (3) $\frac{3}{5}$ (4) 5

Ans. **(2)**

Sol. $f(g(x)) = x$

$\Rightarrow f'(g(x)).\ g'(x) = 1$

Put x = b

$\Rightarrow f'(g(b))\ g'(b) = 1$

$\Rightarrow f'(a) \times 5 = 1$

$\Rightarrow f'(a) = \frac{1}{5}$

13. If $7x + 6y - 2z = 0$

$3x + 4y + 2z = 0$

$x - 2y - 6z = 0$ then which option is correct

$x - 2y - 6z = 0$

(1) no. solution (2) only trivial solution

(3) Infinite non trivial solution for x = 2z (4) Infinite non trivial solution for y = 2z

Ans. **(3)**

Sol.
$$\begin{vmatrix} 7 & 6 & -2 \\ 3 & 4 & 2 \\ 1 & -2 & -6 \end{vmatrix}$$

$= 7(-20) - 6(-20) - 2(-10)$

$= -140 + 120 + 20 = 0$

so infinite non-trivial solution exist

14. Let $x = 2\sin\theta - \sin2\theta$ and

$y = 2\cos\theta - \cos2\theta$

find $\frac{d^2y}{dx^2}$ at $\theta = \pi$

(1) $\frac{3}{8}$ (2) $\frac{3}{2}$ (3) $\frac{5}{8}$ (4) $\frac{7}{8}$

Sol. $\frac{dx}{d\theta} = 2\cos\theta - 2\cos2\theta$

$\frac{dy}{d\theta} = -2\sin\theta + 2\sin2\theta$

$\therefore \frac{dy}{dx} = \frac{\sin2\theta - \sin\theta}{\cos\theta - \cos2\theta}$

$= \frac{2\sin\frac{\theta}{2}.\cos\frac{3\theta}{2}}{2\sin\frac{\theta}{2}\sin\frac{3\theta}{2}} = \cot\frac{3\theta}{2}$

$$\frac{d\ y}{dx^2} = \frac{-3}{2}\text{cosec}^2\frac{3\theta}{2}.\frac{d\theta}{dx}$$

$$\frac{d^2y}{dx^2} = \frac{-\frac{3}{2}\text{cosec}^2\frac{3\theta}{2}}{2(\cos\theta - \cos 2\theta)}$$

$$\frac{d^2y}{dx^2_{(\theta=\pi)}} = -\frac{3}{4(-1-1)} = \frac{3}{8}$$

15. $f(x) : [0, 5] \to R$, $F(x) = \int_0^x x^2\ g(x)$, $f(1) = 3$

$g(x) = \int_1^x f(t)\ dt$ then correct choice is

(1) F(x) has local minimum at x = 1
(2) F(x) has local maximum at x = 1
(3) F(x) has point of inflection at x = 1
(4) F(x) has no critical point

$f(x) : [0, 5] \to R$, $F(x) = \int_0^x x^2\ g(x)$, $f(1) = 3$

Sol. $F'(x) = x^2g(x)$

$\Rightarrow F'(1) = 1.g(1) = 0$ .... (1) $(\because g(1) = 0)$

Now अब $F''(x) = 2xg(x) + x^2g'(x)$

$\Rightarrow F''(x) = 2xg(x) + x^2f(x)$ $(\because g'(x) = f(x))$

$\Rightarrow F''(1) = 0 + 1 \times 3$

$\Rightarrow F''(1) = 3$ .... (2)

From (1) and (2) F(x) has local minimum at x = 1

(1) (2) से F(x), x = 1

16. Let both root of equation $ax^2 - 2bx + 5 = 0$ are α and root of equation $x^2 - 2bx - 10 = 0$ are α and β . Find the value of $\alpha^2 + {}^2$

(1) 20
(2) 25
(3) 15
(4) 30

Ans. **(2)**

Sol. $2\alpha = \frac{2b}{a} \Rightarrow \alpha = \frac{b}{a}$ and $\alpha^2 = \frac{5}{a}$ $\Rightarrow$ $\frac{b^2}{a^2} = \frac{5}{a}$

$\Rightarrow b^2 = 5a$ ..... (i) $(a \neq 0)$

$\alpha + \beta = 2b$ ....(ii)

and तथा $\alpha\beta = -10$ ....(iii)

$\alpha = \frac{b}{a}$ is also root of $x^2 - 2bx - 10 = 0$

$\alpha = \frac{b}{a}$, $x^2 - 2bx - 10 = 0$ का मूल है

$\Rightarrow b^2 - 2ab^2 - 10a^2 = 0$

by (i) से $\Rightarrow 5a - 10a^2 - 10a^2 = 0$

$\Rightarrow 20a^2 = 5a$

$\Rightarrow a = \frac{1}{4}$ and तथा $b^2 = \frac{5}{4}$

$\alpha^2 = 20$ and तथा $\beta^2 = 5$

Now अब $\alpha^2 + \beta^2$

$= 5 + 20$

$= 25$

17. Let $A = \{x : |x| < 2\}$ and $B = \{x : |x - 2| \geq 3\}$ then

माना $A = \{x : |x| < 2\}$ तथा $B = \{x : |x - 2| \geq 3\}$ तब

(1) $A \cap B = [-2, -1]$
(2) $A \cup B = R - (2, 5)$
(3) $A - B = [-1, 2)$
(4) $B - A = R - (-2, 5)$

Ans. **(4)**

Sol. $A = \{ x : x \in (-2, 2) \}$

$B = \{ x : x \in (-\infty, -1] \cup [5, \infty) \}$

$A \cap B = \{ x : x \in (-2, -1] \}$

$A \cup B = \{ x : x \in (-\infty, 2) \cup [5, \infty) \}$

$A - B = \{ x : x \in (-1, 2) \}$

$B - A = \{ x : x \in (-\infty, -2] \cup [5, \infty) \}$

18. Let $x = \sum_{n=0}^{\infty} (-1)^n (\tan\theta)^{2n}$ and $y = \sum_{n=0}^{\infty} (\cos\theta)^{2n}$ where $\theta \in (0, \pi/4)$, then

माना $x = \sum_{n=0}^{\infty} (-1)^n (\tan\theta)^{2n}$ तथा $y = \sum_{n=0}^{\infty} (\cos\theta)^{2n}$ जहाँ $\theta \in (0, \pi/4)$, तब

(1) $x (y + 1) = 1$
(2) $y (1 - x) = 1$
(3) $y (x - 1) = 1$
(4) $y (1 + x) = 1$

Ans. **(2)**

Sol. $y = 1 + \cos^2\theta + \cos^4\theta + ...$

$\Rightarrow y = \frac{1}{1 - \cos^2\theta} \Rightarrow \frac{1}{y} = \sin^2\theta$

$x = \frac{1}{1 - (-\tan^2\theta)} = \frac{1}{\sec^2\theta}$

$\Rightarrow \cos^2\theta = x$

$\Rightarrow \frac{1}{y} + x = 1$

$\Rightarrow y (1 - x) = 1$

19. Let the distance between plane passing through lines $\frac{x+1}{2} = \frac{y-3}{2} = \frac{z+1}{8} = 8$ and $\frac{x+3}{2} = \frac{y+2}{1} = \frac{z-1}{\lambda}$ and plane $23x - 10y - 2z + 48 = 0$ is $\frac{k}{\sqrt{633}}$ then k is equal to

माना सरल रेखाओं $\frac{x+1}{2} = \frac{y-3}{2} = \frac{z+1}{8} = 8$ तथा $\frac{x+3}{2} = \frac{y+2}{1} = \frac{z-1}{\lambda}$ से गुजरने वाले समतल तथा समतल $23x - 10y - 2z + 48 = 0$ के मध्य दूरी $\frac{k}{\sqrt{633}}$ है तब k का मान है–

(1) 1 (2) 2 (3) 3 (4) 4

Ans. **(3)**

Sol. Lines must be intersecting

$\Rightarrow (2s - 1, 3s + 3, 8s - 1) = (2t - 3, t - 2, \lambda t + 1)$

$2s - 1 = 2t - 3,\ 3s + 3 = t - 2,\ 8s - 1 = \lambda t + 1 \qquad \Rightarrow t = -1, s = -2, \lambda = 18$

distance of plane contains given lines from given plane is same as distance between point (–3, –2,1) from given plane.

Required distance equal to $\frac{|-69+20-2+48|}{\sqrt{529+100+4}} = \frac{3}{\sqrt{633}} = \frac{k}{\sqrt{633}} \Rightarrow k = 3$

Sol. रेखाऐं प्रतिच्छेदी होगी

$\Rightarrow (2s - 1, 3s + 3, 8s - 1) = (2t - 3, t - 2, \lambda t + 1)$

$2s - 1 = 2t - 3,\ 3s + 3 = t - 2,\ 8s - 1 = \lambda t + 1 \qquad \Rightarrow t = -1, s = -2, \lambda = 18$

रेखाओं का समाहित करने वाले समतल की दिये हुये समतल से दूरी = बिंदु (–3, –2,1) से दिये गये समतल की दूरी

अभीष्ट दूरी = $\frac{|-69+20-2+48|}{\sqrt{529+100+4}} = \frac{3}{\sqrt{633}} = \frac{k}{\sqrt{633}} \Rightarrow k = 3$

SECTION – 2

- This section contains **FOUR (04)** questions. The answer to each question is **NUMERICAL VALUE** with two digit integer and decimal upto one digit.
- If the numerical value has more than two decimal places **truncate/round-off** the value upto **TWO** decimal places.
  - Full Marks : **+4** If ONLY the correct option is chosen.
  - Zero Marks : **0** In all other cases

खंड 2

- इस खंड में **चार (04)** प्रश्न है। प्रत्येक प्रश्न का उत्तर संख्यात्मक मान **(NUMERICAL VALUE)** हैं, जो द्वि–अंकीय पूर्णांक तथा दशमलव एकल–अंकन में है।
- यदि संख्यात्मक मान में दो से अधिक दशमलव स्थान है , तो संख्यात्मक मान को दशमलव के दो स्थानों तक **ट्रंकेट/राउंड ऑफ (truncate/round-off)** करें।
- अंकन योजना :
  - पूर्ण अंक : **+4** यदि सिर्फ सही विकल्प ही चुना गया है।
  - शून्य अंक : **0** अन्य सभी परिस्थितियों में।

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20. If ${}^{25}C_0 + 5\ {}^{25}C_1 + 9\ {}^{25}C_2$ ……… $101\ {}^{25}C_{25} = 2^{25} k$ find k = ?

यदि ${}^{25}C_0 + 5\ {}^{25}C_1 + 9\ {}^{25}C_2$ ……… $101\ {}^{25}C_{25} = 2^{25} k$ तब k = ?

Ans. **(51)**

Sol. $\sum_{r=0}^{25}(4r+1)^{25}C_r \;= 4\sum_{r=0}^{25} r.^{25}C_r + \sum_{r=0}^{25} {}^{25}C_r$

$= 4\sum_{r=1}^{25} r \times \frac{25}{r}\ {}^{24}C_{r-1} + 2^{25} \;= 100\sum_{r=1}^{25} {}^{24}C_{r-1} + 2^{25}$

$= 100 .2^{24} + 2^{25} = 2^{25}(50 + 1) = 51.2^{25}$

So

21. Let circles $(x - 0)^2 + (y - 4)^2 = k$ and $(x - 3)^2 + (y - 0)^2 = 1^2$ touches each other than find the maximum value of 'k'

Ans. **36.00**

Sol. Two circles touches each other if $C_1 C_2 = |r_1 \pm r_2|$

Distance between $C_2(3, 0)$ and $C_1(0, 4)$ is either $\sqrt{k}+1$ or $\left|\sqrt{k}-1\right|$ $(C_1 C_2 = 5)$

$\Rightarrow \sqrt{k}+1 = 5$ or $\left|\sqrt{k}-1\right| = 5$ $\Rightarrow k = 16$ or $k = 36$ $\Rightarrow$ maximum value of k is 36

22. Let $\left|\vec{a}\right| = \sqrt{3},\ \left|\vec{b}\right| = 5,\ \vec{b}.\vec{c} = 10$ angle between $\vec{b}\ \&\ \vec{c}$ equal to $\frac{\pi}{3}$

If $\vec{a}$ is perpendicular to $\vec{b}\times\vec{c}$ then find the value of $\left|\vec{a}\times\left(\vec{b}\times\vec{c}\right)\right|$

Ans. **30**

Sol. $\vec{b}.\vec{c} = 10 \Rightarrow \left|\vec{b}\right|\ \left|\vec{c}\right|\ \cos\left(\frac{\pi}{3}\right) = 10 \Rightarrow$ $5.\ |\vec{c}|\ .\ \frac{1}{2} = 10 \Rightarrow$ $|\vec{c}| = 4$

Also , $\vec{a}.\left(\vec{b}\times\vec{c}\right) = 0$

$|\vec{a}\times\left(\vec{b}\times\vec{c}\right)| = |\vec{a}||\vec{b}\times\vec{c}|\sin\left(\frac{\pi}{2}\right)$

$\sqrt{3}\times|\vec{b}||\vec{c}|\sin\frac{\pi}{3}\times 1 = \sqrt{3}\times 5\times 4\times\frac{\sqrt{3}}{2} = 30$

23. Number of common terms in both sequence 3, 7, 11, …………407 and 2, 9, 16, ……..905 is

श्रेणियों 3, 7, 11, …………407 तथा 2, 9, 16, ……..905 में उभयनिष्ठ पदों की संख्या है–

Ans. (14)

Sol. First common term = 23

common difference = 7 × 4 = 28

Last term ≤ 407

$\Rightarrow$ 23 + (n –1) × 28 ≤ 407

$\Rightarrow$ (n –1) × 28 ≤ 384

$\Rightarrow$ n ≤ 13.71 + 1

n ≤ 14.71

So n = 14

24. If minimum value of term free from x for $\left(\frac{x}{\sin\theta}+\frac{1}{x\cos\theta}\right)^{16}$ is L_1 in $\theta \in \left[\frac{\pi}{8},\frac{\pi}{4}\right]$ and L_2 in $\theta \in \left[\frac{\pi}{16},\frac{\pi}{8}\right]$ find $\frac{L_2}{L_1}$

Ans. 16

Sol. $T_{r+1} = {}^{16}C_r \left(\frac{x}{\sin\theta}\right)^{16-r}\left(\frac{1}{x\cos\theta}\right)^{r}$

for r = 8 term is free from 'x'

$T_9 = {}^{16}C_8 \frac{1}{\sin^8\theta\cos^8\theta}$

$T_9 = {}^{16}C_8 \frac{2^8}{(\sin 2\theta)^8}$

in $\theta \in \left[\frac{\pi}{8},\frac{\pi}{4}\right]$, $L_1 = {}^{16}C_8\, 2^8$ $\because$ {Min value of L_1 at $\theta = \pi/4$}

in $\theta \in \left[\frac{\pi}{16},\frac{\pi}{8}\right]$, $L_2 = {}^{16}C_8 \frac{2^8}{\left(\frac{1}{\sqrt{2}}\right)^8} = {}^{16}C_8 . 2^8.2^4$ {$\because$ min value of L_2 at $\theta = \pi/8$]

$$\frac{L_2}{L_1} = \frac{{}^{16}C_8.2^8 2^4}{{}^{16}C_8.2^8} = 16$$

Sol. $T_{r+1} = {}^{16}C_r \left(\frac{x}{\sin\theta}\right)^{16-r} \left(\frac{1}{x\cos\theta}\right)^{r}$

r = 8 से 'x'

$T_9 = {}^{16}C_8 \frac{1}{\sin^8\theta\cos^8\theta}$

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$T_9 = {}^{16}C_8 \frac{2^8}{(\sin 2\theta)^8}$

$\theta \in \left[\frac{\pi}{8}, \frac{\pi}{4}\right]$ में, $L_1 = {}^{16}C_8\ 2^8$ $\because$ { L_1 $\theta = \pi/4$ पर}

$\theta \in \left[\frac{\pi}{16}, \frac{\pi}{8}\right]$ में, $L_2 = {}^{16}C_8 \frac{2^8}{\left(\frac{1}{\sqrt{2}}\right)^8} = {}^{16}C_8 . 2^8.2^4$ {$\because$ L_2 $\theta = \pi/8$]

$\frac{L_2}{L_1} = \frac{{}^{16}C_8.2^8 2^4}{{}^{16}C_8.2^8} = 16$