

SECTION – 1

Straight Objective Type (सीधे वस्तुनिष्ठ प्रकार)

This section contains **20 multiple choice questions**. Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which **Only One** is correct.

1. Find the number of solution of $\log_{1/2} |\sin x| = 2 - \log_{1/2} |\cos x|$, $x \in [0, 2\pi]$
 $x \in [0, 2\pi]$ में $\log_{1/2} |\sin x| = 2 - \log_{1/2} |\cos x|$ के हलों की संख्या है—
 (1) 2 (2) 4 (3) 6 (4) 8

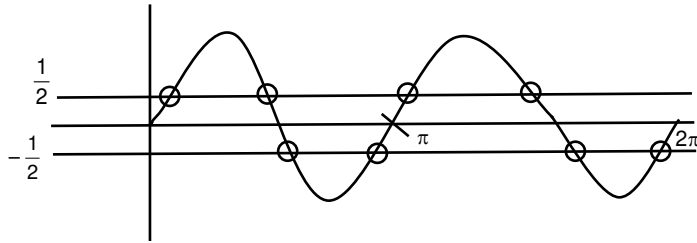
Ans. (4)

Sol. $\log_{1/2} |\sin x| = 2 - \log_{1/2} |\cos x|$

$$\log_{1/2} |\sin x \cos x| = 2$$

$$|\sin x \cos x| = \frac{1}{4}$$

$$\sin 2x = \pm \frac{1}{2}$$



Number of solution हलों की संख्या = 8.

2. If e_1 and e_2 are eccentricities of $\frac{x^2}{18} + \frac{y^2}{4} = 1$ and $\frac{x^2}{9} - \frac{y^2}{4} = 1$, respectively and if the point (e_1, e_2) lies on ellipse $15x^2 + 3y^2 = k$. Then find value of k

- Ans. (3)** (1) 14 (2) 15 (3) 16 (4) 17

Sol. $e_1 = \sqrt{1 - \frac{4}{18}} = \sqrt{\frac{7}{9}} = \frac{\sqrt{7}}{3}$

$$e_2 = \sqrt{1 + \frac{4}{9}} = \sqrt{\frac{13}{9}} = \frac{\sqrt{13}}{3}$$

$$15e_1^2 + 3e_2^2 = k \Rightarrow k = 15\left(\frac{7}{9}\right) + 3\left(\frac{13}{9}\right) \therefore k = 16$$

3. Find integration $\int \frac{1}{x-3} \cdot \frac{1}{x+4} dx$

- (1) $\left(\frac{x-3}{x+4}\right)^{\frac{1}{7}} + c$ (2) $7\left(\frac{x-3}{x+4}\right)^{\frac{1}{7}} + c$ (3) $7\left(\frac{x-3}{x+4}\right)^{\frac{6}{7}} + c$ (4) $7\left(\frac{x+4}{x-3}\right)^{\frac{6}{7}} + c$

Ans. (1)

Sol. $\int \left(\frac{x-3}{x+4}\right)^{-\frac{6}{7}} \cdot \frac{1}{(x+4)^2} dx$

Let $\frac{x-3}{x+4} = t^7$,

$\frac{7}{(x+4)^2} dx = 7t^6 dt$

$\int t^{-6} t^6 dt = t + c$

4. If $\left|\frac{z-i}{z+2i}\right| = 1$, $|z| = \frac{5}{2}$ then value of $|z+3i|$ is

- (1) $\frac{7}{2}$ (2) $\sqrt{10}$ (3) $\sqrt{5}$ (4) $\sqrt{3}$

Ans. (1)

Sol. $x^2 + (y-1)^2 = x^2 + (y+2)^2$

$-2y + 1 = 4y + 4$

$6y = -3 \Rightarrow y = -\frac{1}{2}$

$x^2 + y^2 = \frac{25}{4} \Rightarrow x^2 = \frac{24}{4} = 6$

$\Rightarrow z = \pm \sqrt{6} - \frac{i}{2}$

$|z+3i| = \sqrt{6 + \frac{25}{4}} = \sqrt{\frac{49}{4}}$

$|z+3i| = \frac{7}{2}$

5. $\frac{1}{2^4} \cdot \frac{1}{4^{16}} \cdot \frac{1}{8^{48}} \dots \infty =$

- (1) $\sqrt{2}$ (2) 2 (3) $2^{\frac{1}{4}}$ (4) 1

Sol. $2^{\frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots + \frac{1}{2^n}}$
 $= 2^{\frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots + \frac{1}{2^n}}$ $= \sqrt{2}$

6. Value of $\cos^3 \frac{\pi}{8} \cos \frac{3\pi}{8} + \sin^3 \frac{\pi}{8} \sin \frac{3\pi}{8}$ is

$\cos^3 \frac{\pi}{8} \cos \frac{3\pi}{8} + \sin^3 \frac{\pi}{8} \sin \frac{3\pi}{8}$ का मान है—

- (1) $\frac{1}{2\sqrt{2}}$ (2) $\frac{1}{\sqrt{2}}$ (3) $\frac{1}{2}$ (4) $-\frac{1}{2}$

Ans. (1)

Sol. $\cos^3 \frac{\pi}{8} \left[4\cos^3 \frac{\pi}{8} - 3\cos \frac{\pi}{8} \right] + \sin^3 \frac{\pi}{8} \left[3\sin \frac{\pi}{8} - 4\sin^3 \frac{\pi}{8} \right]$
 $= 4\cos^6 \frac{\pi}{8} - 4\sin^6 \frac{\pi}{8} - 3\cos^4 \frac{\pi}{8} + 3\sin^4 \frac{\pi}{8}$
 $= 4 \left[\left(\cos^2 \frac{\pi}{8} - \sin^2 \frac{\pi}{8} \right) \right] \left[\left(\sin^4 \frac{\pi}{8} + \cos^4 \frac{\pi}{8} + \sin^2 \frac{\pi}{8} \cos^2 \frac{\pi}{8} \right) \right] - 3 \left[\left(\cos^2 \frac{\pi}{8} - \sin^2 \frac{\pi}{8} \right) \right]$
 $= \cos \frac{\pi}{4} \left[4 \left(1 - \sin^2 \frac{\pi}{8} \cos^2 \frac{\pi}{8} \right) - 3 \right] = \frac{1}{\sqrt{2}} \left[1 - \frac{1}{2} \right] = \frac{1}{2\sqrt{2}}$

7. Find the value of $\int_0^{2\pi} \frac{x \sin^8 x}{\sin^8 x + \cos^8 x} dx$

$\int_0^{2\pi} \frac{x \sin^8 x}{\sin^8 x + \cos^8 x} dx$ का मान है—

- (1) π^2 (2) $2\pi^2$ (3) $3\pi^2$ (4) $4\pi^2$

Ans. (1)

Sol. $\int_0^{\pi} \frac{x \sin^8 x}{\sin^8 x + \cos^8 x} + \frac{(2\pi - x) \sin^8 x}{\sin^8 x + \cos^8 x} dx$
 $= \int_0^{\pi} \frac{2\pi \sin^8 x}{\sin^8 x + \cos^8 x} dx$
 $= 2\pi \int_0^{\pi/2} \frac{\sin^8 x}{\sin^8 x + \cos^8 x} + \frac{\cos^8 x}{\sin^8 x + \cos^8 x} dx$
 $= 2\pi \int_0^{\pi/2} 1 dx = 2\pi \times \frac{\pi}{2} = \pi^2$

8. If $f(x) = a + bx + cx^2$ where $a, b, c \in \mathbb{R}$ then $\int_0^1 f(x) dx$ is

(1) $\frac{1}{3} \left(f(1) + f(0) + 2f\left(\frac{1}{2}\right) \right)$

(2) $\frac{1}{6} \left(f(1) + f(0) + 4f\left(\frac{1}{2}\right) \right)$

(3) $\frac{1}{6} \left(f(1) + f(0) - 4f\left(\frac{1}{2}\right) \right)$

(4) $\frac{1}{6} \left(f(1) - f(0) - 4f\left(\frac{1}{2}\right) \right)$

Ans. (2)

Sol. $\int_0^1 (a + bx + cx^2) dx = ax + \frac{bx^2}{2} + \frac{cx^3}{3} \Big|_0^1 = a + \frac{b}{2} + \frac{c}{3}$

$$f(1) = a + b + c$$

$$f(0) = a$$

$$f\left(\frac{1}{2}\right) = a + \frac{b}{2} + \frac{c}{4}$$

$$\text{Now } \frac{1}{6} \left(f(1) + f(0) + 4f\left(\frac{1}{2}\right) \right)$$

$$= \frac{1}{6} \left(a + b + c + a + 4 \left(a + \frac{b}{2} + \frac{c}{4} \right) \right)$$

$$= \frac{1}{6} (6a + 3b + 2c) = a + \frac{b}{2} + \frac{c}{3}$$

9. If number of 5 digit numbers which can be formed without repeating any digit while tenth place of all of the numbers must be 2 is $336k$ find value of k

(1) 8

(2) 7

(3) 6

(4) 5

Ans. (1)

Sol.

			2	
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Number of numbers संख्याओं की संख्या = $8 \times 8 \times 7 \times 6 = 2688 = 336k \Rightarrow k = 8$

10. A (3,-1), B(1,3), C(2,4) are vertices of ΔABC if D is centroid of ΔABC and P is point of intersection of lines $x + 3y - 1 = 0$ and $3x - y + 1 = 0$ then which of the following points lies on line joining D and P
 त्रिभुज ABC के शीर्ष A (3,-1), B(1,3), C(2,4) है यदि D त्रिभुज ABC का केन्द्रक है तथा P रेखाओं $x + 3y - 1 = 0$ तथा $3x - y + 1 = 0$ का प्रतिच्छेद बिन्दु है तो निम्न में से कौनसा बिन्दु D तथा P को जोड़ने वाली रेखा पर स्थित है—
 (1) (-9,-7) (2*) (-9,-6) (3) (9,6) (4) (9,-6)

Ans. (2)

Sol. D (2,2)

Point of intersection P $\left(-\frac{1}{5}, \frac{2}{5}\right)$

equation of line DP

$$8x - 11y + 6 = 0$$

Sol. D (2,2)

$$8x - 11y + 6 = 0$$

11. If $f(x)$ is twice differentiable and continuous function in $x \in [a,b]$ also $f'(x) > 0$ and $f''(x) < 0$ and $c \in (a,b)$ then $\frac{f(c) - f(a)}{c - a}$ is greater than

- (1) $\frac{b-c}{c-a}$ (2) 1 (3) $\frac{a+b}{b-c}$ (4) $\frac{c-a}{b-c}$

Ans. (4)

Sol. Lets use LMVT for $x \in [a,c]$

$$\frac{f(c) - f(a)}{c - a} = f'(\alpha), \alpha \in (a,c)$$

also use LMVT for $x \in [c,b]$

$$\frac{f(b) - f(c)}{b - c} = f'(\beta), \beta \in (c,b)$$

$\because f''(x) < 0 \Rightarrow f'(x)$ is decreasing

$$f'(\alpha) > f'(\beta)$$

$$\frac{f(c) - f(a)}{c - a} > \frac{f(b) - f(c)}{b - c}$$

$$\frac{f(c) - f(a)}{f(b) - f(c)} > \frac{c - a}{b - c} (\because f(x) \text{ is increasing})$$

H_

$$\frac{f(c) - f(a)}{c - a} = f'(\alpha), \alpha \in (a, c)$$

$$f'(\alpha) > f'(\beta)$$

$$\frac{f(c) - f(a)}{c - a} > \frac{f(b) - f(c)}{b - c}$$

$$\frac{f(c) - f(a)}{f(b) - f(c)} > \frac{c - a}{b - c} \quad (\because f(x) \text{ वर्धमान है})$$

12. If plane

$$x + 4y - 2z = 1$$

$$x + 7y - 5z = \beta$$

$$x + 5y + \alpha z = 5$$

intersects in a line ($\mathbb{R} \times \mathbb{R} \times \mathbb{R}$) then $\alpha + \beta$ is equal to

यदि समतल

$$x + 4y - 2z = 1$$

$$x + 7y - 5z = \beta$$

$$x + 5y + \alpha z = 5$$

एक रेखा में ($\mathbb{R} \times \mathbb{R} \times \mathbb{R}$) तो $\alpha + \beta$ बराबर है—

(1) 0

(2) 10

(3) -10

(4) 2

Ans. (2)

Sol. $\Delta = 0 \Rightarrow \begin{vmatrix} 1 & 4 & -2 \\ 1 & 7 & -5 \\ 1 & 5 & \alpha \end{vmatrix} = 0$

$$(7\alpha + 25) - (4\alpha + 10) + (-20 + 14) = 0$$

$$3\alpha + 9 = 0 \Rightarrow \alpha = -3$$

Also तथा $D_z = 0 \Rightarrow \begin{vmatrix} 1 & 4 & 1 \\ 1 & 7 & \beta \\ 1 & 5 & 5 \end{vmatrix} = 0$

$$1(35 - 5\beta) - (15) + 1(4\beta - 7) = 0$$

$$\beta = 13$$

13. For observations x_i given $\sum_{i=1}^{10} (x_i - 5) = 10$ and $\sum_{i=1}^{10} (x_i - 5)^2 = 40$. If mean and variance of observations $(x_1 - 3), (x_2 - 3), \dots, (x_{10} - 3)$ is λ & μ respectively then ordered pair (λ, μ) is
- आंकड़ों x_i के लिये दिया है कि $\sum_{i=1}^{10} (x_i - 5) = 10$ तथा $\sum_{i=1}^{10} (x_i - 5)^2 = 40$ यदि आंकड़ों $(x_1 - 3), (x_2 - 3), \dots, (x_{10} - 3)$ का माध्य λ तथा चरिता μ है तो क्रमित युग्म (λ, μ) है—

- (1) (3, 3) (2) (1, 3) (3) (3, 1) (4) (1, 1)

Ans. (1)

Sol. Mean माध्य $(x_i - 5) = \frac{\sum (x_i - 5)}{10} = 1$

$\therefore \lambda = \{\text{Mean माध्य } (x_i - 5)\} + 2 = 3$

$\mu = \text{var चरिता } (x_i - 5) = \frac{\sum (x_i - 5)^2}{10} - \frac{\sum (x_i - 5)^2}{10} = 3$

14. In a bag there are 20 cards 10 names A and another 10 names B. Cards are drawn randomly one by one with replacement then find probability that second A comes before third B.

- (1) $\frac{13}{16}$ (2) $\frac{11}{16}$ (3) $\frac{7}{16}$ (4) $\frac{9}{16}$

Ans. (2)

Sol. AA + ABA + BAA + ABBA + BBAA + BABA

$= \frac{1}{4} + \frac{1}{8} + \frac{1}{8} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} = \frac{11}{16}$

15. The negation of ' $\sqrt{5}$ is an integer or 5 is an irrational number' is

- (1) $\sqrt{5}$ is an integer and 5 is not an irrational Number
 (2) $\sqrt{5}$ is not an integer and 5 is an irrational Number
 (3) $\sqrt{5}$ is not an integer or 5 is not an irrational Number
 (4) $\sqrt{5}$ is not an integer and 5 is not an irrational Number

Ans. (4)

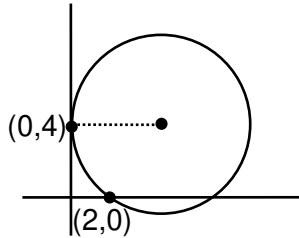
Sol. $\sqrt{5}$ is not an integer and 5 is not an irrational Number $\sim (p \vee q) = \sim p \wedge \sim q$

16. If a circle touches y-axis at (0, 4) and passes through (2, 0) then which of the following can not be the

- (1) $3x + 4y - 6 = 0$ (2) $3x + 4y - 24 = 0$ (3) $4x - 3y - 17 = 0$ (4) $4x + 3y - 6 = 0$

Ans. (1)

Sol.



equation of family of circle

$$(x - 0)^2 + (y - 4)^2 + \lambda x = 0$$

\Rightarrow passes गुजरता है (2, 0)

$$4 + 16 + 2\lambda = 0 \Rightarrow \lambda = -10$$

$$x^2 + y^2 - 10x - 8y + 16 = 0$$

centre केंद्र (5, 4). $R = \sqrt{25 + 16 - 16} = 5$

17. If $f'(x) = \tan^{-1}(\sec x + \tan x)$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and $f(0) = 0$ then the value of $f(1)$ is

- (1) $\frac{\pi+1}{4}$ (2) $\frac{\pi-1}{4}$ (3) $\frac{\pi+1}{2}$ (4) 0

Ans. (1)

Sol. $f'(x) = \tan^{-1}(\sec x + \tan x) = \tan^{-1}\left(\frac{1 + \sin x}{\cos x}\right) = \tan^{-1}\left(\frac{1 - \cos\left(\frac{\pi}{2} + x\right)}{\sin\left(\frac{\pi}{2} + x\right)}\right) = \tan^{-1}\left(\frac{2\sin^2\left(\frac{\pi}{4} + \frac{x}{2}\right)}{2\sin\left(\frac{\pi}{4} + \frac{x}{2}\right)\cos\left(\frac{\pi}{4} + \frac{x}{2}\right)}\right)$

$$= \tan^{-1}\left(\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right) = \frac{\pi}{4} + \frac{x}{2}$$

$$(f'(x))dx = \frac{\pi}{4} + \frac{x}{2} dx$$

$$f(x) = \frac{\pi}{4}x + \frac{x^2}{4} + c$$

$$f(0) = c = 0 \Rightarrow f(x) = \frac{\pi}{4}x + \frac{x^2}{4}$$

18. A sphere of 10cm radius has a uniform thickness of ice around it. Ice is melting at rate $50\text{cm}^3/\text{min}$ when thickness is 5cm then rate of change of thickness

- (1) $\frac{1}{36\pi}$ (2) $\frac{1}{18\pi}$ (3) $\frac{1}{9\pi}$ (4) $\frac{1}{12\pi}$

Ans. (2)

Sol. Let thickness माना मोटाई = x cm

Total volume कुल आयतन $v = \frac{4}{3}\pi(10 + x)^3$

$$\frac{dv}{dt} = 4\pi(10 + x)^2 \frac{dx}{dt} \dots\dots\dots(i)$$

Given दिया है $\frac{dv}{dt} = 50\text{cm}^3/\text{min}$

At $x = 5\text{cm}$

$$50 = 4\pi(10 + 5)^2 \frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{1}{18\pi} \text{ cm/min}$$

19. Find number of real roots of equation $e^{4x} + e^{3x} - 4e^{2x} + e^x + 1 = 0$ is

- (1) 1 (2) 2 (3) 3 (4) 4

Ans. (1)

Sol. Let $e^x = t \in (0, \infty)$

Given equation

$$t^4 + t^3 - 4t^2 + t + 1 = 0$$

$$t^2 + t - 4 + \frac{1}{t} + \frac{1}{t^2} = 0$$

$$\left(t^2 + \frac{1}{t^2}\right) + \left(t + \frac{1}{t}\right) - 4 = 0$$

Let $t + \frac{1}{t} = \alpha$

$$(\alpha^2 - 2) + \alpha - 4 = 0$$

$$\alpha^2 + \alpha - 6 = 0$$

$$\alpha^2 + \alpha - 6 = 0$$

$$\alpha = -3, 2 \quad \Rightarrow \quad \alpha = 2 \quad \Rightarrow \quad e^x + e^{-x} = 2$$

$x = 0$ only solution

20. If $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 3 & 4 \\ 1 & -1 & 3 \end{bmatrix}$, $B = \text{adj}(A)$ and $C = 3A$ then $\frac{|\text{adj}B|}{|C|}$ is

यदि $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 3 & 4 \\ 1 & -1 & 3 \end{bmatrix}$, $B = \text{adj}(A)$ तथा $C = 3A$ तब $\frac{|\text{adj}B|}{|C|}$ बराबर है-

(1) 8

(2) 4

(3) 2

(4) 16

Ans. (1)

Sol. $|A| = \begin{vmatrix} 1 & 1 & 2 \\ 1 & 3 & 4 \\ 1 & -1 & 3 \end{vmatrix} = ((9+4) - 1(3-4) + 2(-1-3))$

$$= 13 + 1 - 8 = 6$$

$$|\text{adj}B| = |\text{adjadj}A| = |A|^{(n-1)^2} = |A|^4 = (36)^2$$

$$|C| = |3A| = 3^3 \times 6$$

$$\frac{|\text{adj}B|}{|C|} = \frac{36 \times 36}{3^3 \times 6} = 8$$

SECTION - 2

- ❖ This section contains **FIVE (05)** questions. The answer to each question is **NUMERICAL VALUE** with two digit integer and decimal upto one digit.
- ❖ If the numerical value has more than two decimal places **truncate/round-off** the value upto **TWO** decimal places.
 - Full Marks : **+4** If **ONLY** the correct option is chosen.
 - Zero Marks : **0** In all other cases

21. $(1+x) \frac{dy}{dx} = ((1+x)^2 + (y-3))$, If $y(2) = 0$ then $y(3) = ?$

$(1+x) \frac{dy}{dx} = ((1+x)^2 + (y-3))$, यदि $y(2) = 0$ तो $y(3) = ?$

Ans. 3

Sol. $\frac{dy}{dx} = (1+x) + \left(\frac{y-3}{1+x}\right)$

$$\frac{dy}{dx} - \frac{1}{(1+x)}y = (1+x) - \frac{3}{(1+x)}$$

$$\text{I.F.} = e^{-\int \frac{1}{1+x} dx} = \frac{1}{(1+x)}$$

$$\therefore \frac{d}{dx} \left(\frac{y}{1+x} \right) = 1 - \frac{3}{(1+x)^2}$$

$$\frac{y}{1+x} = x + 3(1+x)^{-1} + c$$

$$y = (1+x) \left[x + \frac{3}{(1+x)} + c \right]$$

$$\therefore \text{ at } x = 2, \quad y = 0$$

$$\therefore 0 = 3(2+1+c) \Rightarrow c = -3$$

$$\therefore \text{ at } x = 3, \quad y = 3$$

$$22. \quad f(x) = \begin{cases} \frac{\sin(a+2)x + \sin x}{x}; & x < 0 \\ b & ; x = 0 \\ \frac{(x+3x^2)^{\frac{1}{3}} - x^{\frac{1}{3}}}{\frac{4}{x^3}}; & x > 0 \end{cases}$$

Function is continuous at $x = 0$, find $a + 2b$.

$$f(x) = \begin{cases} \frac{\sin(a+2)x + \sin x}{x}; & x < 0 \\ b & ; x = 0 \\ \frac{(x+3x^2)^{\frac{1}{3}} - x^{\frac{1}{3}}}{\frac{4}{x^3}}; & x > 0 \end{cases}$$

Ans. 0

Sol. LHL = $a + 3$

$$f(0) = b$$

$$\text{RHL} = \lim_{h \rightarrow 0} \left(\frac{(1+3h)^{\frac{1}{3}} - 1}{h} \right) = 1$$

$$\therefore a = -2$$

$$b = 1$$

$$\therefore a + 2b = 0$$

23. Find the coefficient of x^4 in $(1 + x + x^2)^{10}$

Ans. 615

Sol. General term $\frac{10!}{\alpha!\beta!\gamma!} x^{\beta+2\gamma}$
 for coefficient of $x^4 \Rightarrow \beta + 2\gamma = 4$
 $\gamma = 0, \beta = 4, \alpha = 6 \Rightarrow \frac{10!}{6!4!0!} = 210$
 $\gamma = 1, \beta = 2, \alpha = 7 \Rightarrow \frac{10!}{7!2!1!} = 360$
 $\gamma = 2, \beta = 0, \alpha = 8 \Rightarrow \frac{10!}{8!0!2!} = 45$
 Total कुल = 615

24. If $\vec{P} = (a+1)\hat{i} + a\hat{j} + a\hat{k}$
 $\vec{Q} = a\hat{i} + (a+1)\hat{j} + a\hat{k}$
 $\vec{R} = a\hat{i} + a\hat{j} + (a+1)\hat{k}$
 and $\vec{P}, \vec{Q}, \vec{R}$ are coplanar vectors and $3(\vec{P} \cdot \vec{Q})^2 - \lambda |\vec{R} \times \vec{Q}|^2 = 0$ then value of λ is
 $\vec{P} = (a+1)\hat{i} + a\hat{j} + a\hat{k}$
 $\vec{Q} = a\hat{i} + (a+1)\hat{j} + a\hat{k}$
 $\vec{R} = a\hat{i} + a\hat{j} + (a+1)\hat{k}$

Ans. 1

Sol. $\begin{vmatrix} a+1 & a & a \\ a & a+1 & a \\ a & a & a+1 \end{vmatrix} = 0 \Rightarrow a+1+a+a=0 \Rightarrow a = -\frac{1}{3}$

$$\vec{P} = \frac{2}{3}\hat{i} - \frac{1}{3}\hat{j} - \frac{1}{3}\hat{k}$$

$$\vec{Q} = \frac{1}{3}(-\hat{i} + 2\hat{j} - \hat{k})$$

$$\vec{R} = \frac{1}{3}(-\hat{i} - \hat{j} + 2\hat{k})$$

$$\vec{P} \cdot \vec{Q} = \frac{1}{9}(-2 - 2 + 1) = -\frac{1}{3}$$

$$\vec{R} \times \vec{Q} = \frac{1}{9} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{vmatrix} = \frac{1}{9} (i(4-1) - j(-2-1) + k(1+2))$$

$$= \frac{1}{9} (3i + 3j + 3k) = \frac{i+j+k}{3}$$

$$|\vec{R} \times \vec{Q}| = \frac{1}{3} \sqrt{3} \Rightarrow |\vec{R} \times \vec{Q}|^2 = \frac{1}{3}$$

$$3(\vec{P} \cdot \vec{Q})^2 - \lambda |\vec{R} \times \vec{Q}|^2 = 0$$

$$3 \cdot \frac{1}{9} - \lambda \cdot \frac{1}{3} = 0 \Rightarrow \lambda = 1$$

25. If points A (2, 4, 0), B(3, 1, 8), C(3, 1, -3), D(7, -3, 4) are four points then projection of line segment AB on line CD.

Sol. $\vec{AB} = (\hat{i}) - (3\hat{j}) + 8\hat{k}$

$$\vec{CD} = 4\hat{i} - 4\hat{j} + 7\hat{k}$$

$$(\vec{AB} \cdot \vec{CD}) = \frac{4 + 12 + 56}{\sqrt{16 + 16 + 49}} = \frac{72}{9} = 8$$