PART : MATHEMATICS

SECTION – 1

Straight Objective Type

This section contains **22 multiple choice questions.** Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which **Only One** is correct.

1. Let $\int \frac{\cos x\, dx}{\sin^3 x(1+\sin^6 x)^{\frac{2}{3}}} = f(x)\ (1+\sin^6 x)^{\frac{1}{\lambda}} + c$ then find value of $\lambda f\left(\frac{\pi}{3}\right)$

(1) 4 (2) –2 (3)8 (4) –4

Ans. (2)

Sol. sin x = t

cos x dx = dx

$I = \int \frac{dt}{t^3(1+t^6)^{\frac{2}{3}}} = \int \frac{dt}{t^7\left(1+\frac{1}{t^6}\right)^{\frac{2}{3}}}$

Put $1+\frac{1}{t^6} = r^3 \qquad \Rightarrow \frac{dt}{t^7} = \frac{-1}{2} r^2$ dr

$-\frac{1}{2}\int \frac{r^2\,dr}{r^2} = -\frac{1}{2}\ r + c = -\frac{1}{2}\left(\frac{\sin^6 x+1}{\sin^6 x}\right)^{\frac{1}{3}} + c = -\frac{1}{2\sin^2 x}(1+\sin^6 x)^{\frac{1}{3}} + c$

$f(x) = -\frac{1}{2}\text{cosec}^2 x$ and $\lambda = 3$

$\lambda f\left(\frac{\pi}{3}\right) = -2$

2. If y(x) is a solution of differential equation $\sqrt{1-x^2}\frac{dy}{dx} + \sqrt{1-y^2} = 0$, such that $y\left(\frac{1}{2}\right) = \frac{\sqrt{3}}{2}$, then

(1) $y\left(\frac{1}{\sqrt{2}}\right) = -\frac{1}{\sqrt{2}}$ (2) $y\left(\frac{1}{\sqrt{2}}\right) = \frac{\sqrt{3}}{2}$ (3) $y\left(\frac{1}{\sqrt{2}}\right) = \frac{1}{\sqrt{2}}$ (4) $y\left(\frac{1}{2}\right) = \frac{1}{2}$

Ans. **(3)**

Sol. $\frac{dy}{\sqrt{1-y^2}} + \frac{dx}{\sqrt{1-x^2}} = 0 \quad \Rightarrow \quad \sin^{-1}y + \sin^{-1}x = c$

At $x = \frac{1}{2}$, $y = \frac{\sqrt{3}}{2}$ $\Rightarrow$ $c = \frac{\pi}{2}$ $\Rightarrow$ $\sin^{-1}y = \cos^{-1}x$

Hence अतः $y\left(\frac{1}{\sqrt{2}}\right) = \sin\left(\cos^{-1}\frac{1}{\sqrt{2}}\right) = \frac{1}{\sqrt{2}}$

3. $\lim_{x\to 0}\left(\frac{3x^2+2}{7x^2+2}\right)^{\frac{1}{x^2}}$ is equal to

(1) e^{-2} (2) e^{2} (3) $e^{2/7}$ (4) $e^{3/7}$

Ans. **(1)**

Sol. Let $L = \lim_{x\to 0}\left(\frac{3x^2+2}{7x^2+2}\right)^{\frac{1}{x^2}} = e^{\lim_{x\to 0}\frac{1}{x^2}\left\{\frac{3x^2+2}{7x^2+2}-1\right\}} = e^{\lim_{x\to 0}\frac{1}{x^2}\left\{\frac{-4x^2}{7x^2+2}\right\}} = e^{\frac{-4}{2}} = e^{-2}$

4. In a bag there are 5 red balls, 3 white balls and 4 black balls. Four balls are drawn from the bag. Find the number of ways of in which at most 3 red balls are selected

(1) 450 (2) 360 (3) 490 (4) 510

Ans. **(3)**

Sol. 0 Red, 1Red, 2 Red, 3 Red

Number of ways = ${}^7C_4 + {}^5C_1.{}^7C_3 + {}^5C_2.{}^7C_2 + {}^5C_3.{}^7C_1 = 35 + 175 + 210 + 70 = 490$

5. Let $f(x) = \{(\sin(\tan^{-1}x) + \sin(\cot^{-1}x)\}^2 - 1$ where $|x| > 1$ and $\frac{dy}{dx} = \frac{1}{2}\frac{d}{dx}\left(\sin^{-1}f(x)\right)$.

If $y\left(\sqrt{3}\right) = \frac{\pi}{6}$ then $y\left(-\sqrt{3}\right) =$

(1) $\frac{5\pi}{6}$ (2) $\frac{-\pi}{6}$ (3) $\frac{\pi}{3}$ (4) $\frac{2\pi}{3}$

Ans. **(2)**

Sol. $2y = \sin^{-1}f(x) + C = \sin^{-1}(\sin(2\tan^{-1}x)) + C$ $\Rightarrow$ $2\left(\frac{\pi}{6}\right) = \sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right) + C$

$\frac{\pi}{3} = \frac{\pi}{3} + C$ $\therefore$ $C = 0$

for $x = -\sqrt{3}$, $2y = \sin^{-1}\left(\sin\left(\frac{-2\pi}{6}\right)\right) + 0$ $\Rightarrow$ $2y = \frac{-\pi}{3}$

$\left(y = \frac{-\pi}{6}\right)$

6. If $2^{1-x} + 2^{1+x}$, $f(x)$, $3^x + 3^{-x}$ are in A.P. then minimum value of $f(x)$ is

(1) 1 (2) 2 (3) 3 (4) 4

Ans. **(3)**

Sol. $f(x) = \left(\frac{2^{1-x} + 2^{1+x} + 3^{x} + 3^{-x}}{2}\right)$

Using AM ≥ GM

$f(x) \geq 3$

7. Which of the following is tautology

(1) $(p \wedge (p \rightarrow q)) \rightarrow q$ (2) $q \rightarrow p \wedge (p \rightarrow q)$

(3) $p \vee (p \wedge q)$ (4) $(p \wedge (p \vee q)$

Ans. **(1)**

Sol.

p	q	p→ q	p∧ (p → q)	(p ∧(p →q)) → q	q → p∧(p→q)	p ∧ q	p ∨ (p ∧q)	p ∨ q	p∧ (p∨ q)
T	T	T	T	T	T	T	T	T	T
T	F	F	F	T	T	F	T	T	T
F	T	T	F	T	F	F	F	T	F
F	F	T	F	T	T	F	F	F	F

8. A is a 3 × 3 matrix whose elements are from the set {–1, 0, 1}. Find the number of matrices A such that $tr(AA^T) = 3$. Where tr(A) is sum of diagonal elements of matrix A.

(1) 572 (2) 612 (3) 672 (4) 682

Ans. **(3)**

Sol. Let $A = [a_{ij}]_{3\times3}$

$tr(AA^T) = 3$

$a_{11}^2 + a_{12}^2 + a_{13}^2 + a_{21}^2 + \text{..........} + a_{33}^2 = 3$

possible cases

$$\left.\begin{matrix} 0,0,0,0,0,0,1,1,1 & \rightarrow 1 \\ 0,0,0,0,0,0,-1,-1,-1 & \rightarrow 1 \\ 0,0,0,0,0,0,1,1,-1 & \rightarrow 3 \\ 0,0,0,0,0,0,-1,1,-1 & \rightarrow 3 \end{matrix}\right\} {}^9C_6 \times 8 = 84 \times 8 = 672$$

9. Mean and standard deviations of 10 observations are 20 and 2 respectively. If p (p ≠ 0) is multiplied to each observation and then q (q ≠ 0) is subtracted then new mean and standard deviation becomes half of original value . Then find q

(1) –10 (2) –20 (3) –5 (4) 10

Ans. (2)

Sol. If each observation is multiplied with p & then q is subtracted

New mean $\bar{x}_1 = p\bar{x} - q$

$\Rightarrow 10 = p(20) - q$ .....(1)

and new standard deviations

$\sigma_2 = |p|\,\sigma_1 \quad \Rightarrow 1 = |p|\,(2) \quad \Rightarrow |p| = \frac{1}{2} \quad \Rightarrow p = \pm\frac{1}{2}$

If $p = \frac{1}{2}$

then q = 0 (from equation (1))

If $p = -\frac{1}{2}$

$q = -20$

10. If maximum value of $^{19}C_p$ is a, $^{20}C_q$ is b, $^{21}C_r$ is c, then relation between a, b, c is

(1) $\frac{a}{11} = \frac{b}{22} = \frac{c}{42}$ (2) $\frac{a}{22} = \frac{b}{11} = \frac{c}{42}$ (3) $\frac{a}{22} = \frac{b}{42} = \frac{c}{11}$ (4) $\frac{a}{21} = \frac{b}{11} = \frac{c}{22}$

Ans. **(1)**

Sol. We know ^nC_r is max at middle term

$a = {}^{19}C_p = {}^{19}C_{10} = {}^{19}C_9$

$b = {}^{20}C_q = {}^{20}C_{10}$

$c = {}^{21}C_6 = {}^{21}C_{10} = {}^{21}C_{11}$

$$\frac{a}{^{19}C_9} = \frac{b}{\frac{20}{10}\cdot{}^{19}C_9} = \frac{c}{\frac{21}{11}\cdot\frac{20}{10}\,{}^{19}C_9}$$

$$\frac{a}{1} = \frac{b}{2} = \frac{c}{42/11}$$

$$\frac{a}{11} = \frac{b}{22} = \frac{c}{42}$$

11. Let P(A) = $\frac{1}{3}$, P(B) = $\frac{1}{6}$ where A and B are independent events then

(1) $P\left(\frac{A}{B}\right)=\frac{1}{6}$ (2) $P\left(\frac{A}{B'}\right)=\frac{1}{3}$ (3) $P\left(\frac{A}{B'}\right)=\frac{2}{3}$ (4) $P\left(\frac{A}{B}\right)=\frac{5}{6}$

Ans. **(2)**

Sol. A & B are independent events so

$$P\left(\frac{A}{B'}\right)=\frac{1}{3}$$

12. Let $f(x)=\frac{8^{2x}-8^{-2x}}{8^{2x}+8^{-2x}}$ then inverse of f(x) is

(1) $\frac{1}{4}\log_8\left(\frac{1+x}{1-x}\right)$ (2) $\frac{1}{2}\log_8\left(\frac{1-x}{1+x}\right)$ (3) $\frac{1}{4}\log_8\left(\frac{1-x}{1+x}\right)$ (4) $\frac{1}{2}\log_8\left(\frac{1+x}{1-x}\right)$

Ans. **(1)**

Sol. $y=\frac{8^{2x}-8^{-2x}}{8^{2x}+8^{-2x}}$

$$\frac{1+y}{1-y}=\frac{8^{2x}}{8^{-2x}}$$

$$8^{4x}=\frac{1+y}{1-y}$$

$$4x=\log_8\left(\frac{1+y}{1-y}\right)$$

$$x=\frac{1}{4}\log_8\left(\frac{1+y}{1-y}\right)$$

$$f^{-1}(x)=\frac{1}{4}\log_8\left(\frac{1+x}{1-x}\right)$$

13. Roots of the equation $x^2 + bx + 45 = 0$, $b \in R$ lie on the curve $|z + 1| = 2\sqrt{10}$, where z is a complex number then

(1) $b^2 + b = 12$ (2) $b^2 - b = 30$ (3) $b^2 - b = 36$ (4) $b^2 + b = 30$

Ans. (2)

Sol. Let $z = \alpha \pm i\beta$ be roots of the equation

So $2\alpha = -b$ and $\alpha^2 + \beta^2 = 45$, $(\alpha + 1)^2 + \beta^2 = 40$

So $(\alpha + 1)^2 - \alpha^2 = -5$

$\Rightarrow$ $2\alpha + 1 = -5$ $\Rightarrow$ $2\alpha = -6$

so b = 6

hence $b^2 - b = 30$

14. For $f(x) = \ell n\left(\frac{x^2+\alpha}{7x}\right)$. Rolle's theorem is applicable on [3, 4], the value of f"(c) is equal to

(1) $\frac{1}{12}$ (2) $\frac{-1}{12}$ (3) $\frac{1}{6}$ (4) $\frac{-1}{6}$

Sol. $f(3) = f(4) \Rightarrow \alpha = 12$

$f'(x) = \frac{x^2-12}{x(x^2+12)}$

$\therefore$ $f'(c) = 0$

$\therefore$ $c = \sqrt{12}$

$\therefore$ $f''(c) = \frac{1}{12}$

15. Let $f(x) = x\cos^{-1}(\sin(-|x|))$, $x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ then

(1) $f'(0) = -\frac{\pi}{2}$

(2) $f'(x)$ is not defined at $x = 0$

(3) $f'(x)$ is increasing in $\left(\frac{-\pi}{2}, 0\right)$ and $f'(x)$ is decreasing in $\left(0, \frac{\pi}{2}\right)$

(4) $f'(x)$ is decreasing in $\left(\frac{-\pi}{2}, 0\right)$ and $f'(x)$ is increasing in $\left(0, \frac{\pi}{2}\right)$

Sol. $f'(x) = x(\pi - \cos^{-1}(\sin|x|))$

$= x\left(\pi - \left(\frac{\pi}{2} - \sin^{-1}(\sin|x|)\right)\right)$

$= x\left(\frac{\pi}{2} + |x|\right)$

$$f(x) = \begin{cases} x\left(\frac{\pi}{2} + x\right) & x \geq 0 \\ x\left(\frac{\pi}{2} - x\right) & x < 0 \end{cases}$$

$$f'(x) = \begin{cases} \frac{\pi}{2} + 2x & x \geq 0 \\ \frac{\pi}{2} - 2x & x < 0 \end{cases}$$

$f'(x)$ is increasing in $\left(0, \frac{\pi}{2}\right)$ and decreasing in $\left(\frac{-\pi}{2}, 0\right)$

16. Let P be a point on $x^2 = 4y$. The segment joining A (0,–1) and P is divided by point Q in the ration 1:2, then locus of point Q is

(1) $9x^2 = 3y + 2$ (2) $9x^2 = 12y + 8$ (3) $9y^2 = 12x + 8$ (4) $9y^2 = 3x + 2$

Ans. **(2)**

Sol. Let point P be $(2t, t^2)$ and Q be (h, k).

$h = \frac{2t}{3}$, $k = \frac{-2 + t^2}{3}$

Hence locus is $3k + 2 = \left(\frac{3h}{2}\right)^2 \Rightarrow 9x^2 = 12y + 8$

17. Ellipse $2x^2 + y^2 = 1$ and $y = mx$ meet a point A in first quadrant. Normal to the ellipse at P meets x-axis at $\left(-\frac{1}{3\sqrt{2}}, 0\right)$ and y-axis at $(0, \beta)$, then $|\beta|$ is

(1) $\frac{2}{\sqrt{3}}$ (2) $\frac{2\sqrt{2}}{3}$ (3) $\frac{\sqrt{2}}{3}$ (4) $\frac{2}{3}$

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Ans. (3)

Sol. Let P be (x_1, y_1)

Equation of normal at P is $\frac{x}{2x_1} - \frac{y}{y_1} = -\frac{1}{2}$

It passes through $\left(-\frac{1}{3\sqrt{2}}, 0\right) \Rightarrow \frac{-1}{6\sqrt{2}x_1} = -\frac{1}{2} \quad \Rightarrow x_1 = \frac{1}{3\sqrt{2}}$

So $y_1 = \frac{2\sqrt{2}}{3}$ (as P lies in I^{st} quadrant)

So $\beta = \frac{y_1}{2} = \frac{\sqrt{2}}{3}$

18. If $y^2 = ax$ and $x^2 = ay$ intersect at A & B. Area bounded by both curves is bisected by line $x = b$(given $a > b > 0$). Area of triangle formed by line AB, $x = b$ and x-axis is $\frac{1}{2}$. Then

(1) $a^6 - 12a^3 - 4 = 0$ (2) $a^6 + 12a^3 - 4 = 0$

(3) $a^6 - 12a^3 + 4 = 0$ (4) $a^6 + 12a^3 + 4 = 0$

Ans. **(3)**

Sol. $\int_0^b \left(\sqrt{a}x^{\frac{1}{2}} - \frac{x^2}{a}\right)dx = \frac{a^2}{6}$

$\Rightarrow \quad \frac{2}{3}\sqrt{a}\, b^{\frac{3}{2}} - \frac{b^3}{3a} = \frac{a^2}{6}$ ....(i)

also area of Δ OQR $\frac{1}{2}$

$\frac{1}{2}b^2 = \frac{1}{2} \quad \Rightarrow \quad b = 1$

Put in (i)

$\Rightarrow \quad 4a\sqrt{a} - 2 = a^3$

$\Rightarrow \quad a^6 + 4a^3 + 4 = 16a^3$

$\Rightarrow \quad a^6 - 12a^3 + 4 = 0$

19. Let ABC is a triangle whose vertices are A(1, –1), B(0, 2), C(x′, y′)and area of ΔABC is 5 and C(x′, y′) lie on $3x + y - 4\lambda = 0$, then

(1) $\lambda = 3$ (2) $\lambda = -3$ (3) $\lambda = 4$ (4) $\lambda = 2$

Ans. (1)

Sol. $D = \frac{1}{2}\begin{vmatrix} 0 & 2 & 1 \\ 1 & -1 & 1 \\ x' & y' & 1 \end{vmatrix}$

$-2(1 - x') + (y' + x') = \pm 10$

$-2 + 2x' + y' + x' = \pm 10$

$3x' + y' = 12$ or या $3x' + y' = -8$

$\lambda = 3, -2$

20. The system of equation $3x + 4y + 5z = \mu$

$x + 2y + 3z = 1$

$4x + 4y + 4z = \delta$

is inconsistent, then (δ, μ) can be

(1) (4, 6) (2) (3, 4) (3) (4, 3) (4) (1, 0)

Ans. **(3)**

Sol. Note $D = \begin{vmatrix} 3 & 4 & 5 \\ 1 & 2 & 3 \\ 4 & 4 & 4 \end{vmatrix}$ $(R_3 \to R_3 - 2R_1 + 3R_2)$

$$= \begin{vmatrix} 3 & 4 & 5 \\ 1 & 2 & 3 \\ 0 & 0 & 0 \end{vmatrix} = 0$$

Now let $P_3 \equiv 4x + 4y + 4z - \delta = 0$. If the system has solutions it will have infinite solution,
so $P_3 \equiv \alpha P_1 + \beta P_2$
Hence $3\alpha + \beta = 4$ & $4\alpha + 2\beta = 4 \Rightarrow \alpha = 2$ & $\beta = -2$
So for infinite solution $2\mu - 2 = \delta \Rightarrow$ for $2\mu \neq \delta + 2$
system inconsistent

21. Shortest distance between the lines $\frac{x-3}{1} = \frac{y-8}{4} = \frac{z-3}{22}$, $\frac{x+3}{1} = \frac{y+7}{1} = \frac{z-6}{7}$ is

(1) $3\sqrt{30}$ (2) $2\sqrt{30}$ (3) $\sqrt{30}$ (4) $4\sqrt{30}$

Ans. **(1)**

Sol. $\overrightarrow{AB} = 6\hat{i} + 15\hat{j} + 3\hat{k}$

$\vec{p} = \hat{i} + 4\hat{j} + 22\hat{k}$

$\vec{q} = \hat{i} + \hat{j} + 7\hat{k}$

$$\vec{p} \times \vec{q} = \begin{vmatrix} i & j & k \\ 1 & 4 & 22 \\ 1 & 1 & 7 \end{vmatrix} = 6\hat{i} + 15\hat{j} - 3\hat{k}$$

$$\text{S.D.} = \frac{\left|\overrightarrow{AB}.(\vec{p} \times \vec{q})\right|}{|\vec{p} \times \vec{q}|} = \frac{|36 + 225 + 9|}{\sqrt{36 + 225 + 9}} = 3\sqrt{30}$$

22. If volume of parallelopiped whose there coterminous edges are $\vec{u} = \hat{i} + \hat{j} + \lambda\hat{k}$, $\vec{v} = 2\hat{i} + \hat{j} + \hat{k}$ & $\vec{w} = \hat{i} + \hat{j} + 3\hat{k}$ is 1 cubic unit then cosine of angle between $\vec{u}$ and $\vec{v}$ is

(1) $\frac{7}{3\sqrt{10}}$ (2) $\frac{7}{6\sqrt{3}}$ (3) $\frac{5}{3\sqrt{3}}$ (4) $\frac{5}{7}$

Sol. $\pm 1 = \begin{vmatrix} 1 & 1 & \lambda \\ 1 & 1 & 3 \\ 2 & 1 & 1 \end{vmatrix} \Rightarrow = -\lambda + 3 = \pm 1 \Rightarrow \lambda = 2 \text{ or } \lambda = 4$

For $\lambda = 4$

$\cos\theta = \frac{2+1+4}{\sqrt{6}\sqrt{18}} = \frac{7}{6\sqrt{3}}$

SECTION – 2

- This section contains **FIVE (03)** questions. The answer to each question is **NUMERICAL VALUE** with two digit integer and decimal upto one digit.
- If the numerical value has more than two decimal places **truncate/round-off** the value upto **TWO** decimal places.
  - Full Marks : **+4** If ONLY the correct option is chosen.
  - Zero Marks : **0** In all other cases

23. Find the sum $\sum_{k=1}^{20}(1+2+3+......+k)$

Ans. **1540**

Sol. $= \sum_{k=1}^{20}\frac{k(k+1)}{2}$

$= \frac{1}{2}\sum_{k=1}^{20}(k^2+k)$

$= \frac{1}{2}\left[\frac{20(21)(41)}{6} + \frac{20(21)}{2}\right]$

$= \frac{1}{2}\left[\frac{420\times 41}{6} + \frac{20\times 21}{2}\right]$

$= \frac{1}{2}[2870 + 210]$

$= 1540$

24. If normal at P on the curve $y^2 - 3x^2 + y + 10 = 0$ passes through the point (0, 3/2) then slope of tangent at P is n. The value of |n| is equal to

Ans. **4**

Sol. $P \equiv (x_1, y_1)$

$2yy' - 6x + y' = 0 \Rightarrow y' = \left(\frac{6x_1}{1+2y_1}\right)$

$$\left(\frac{\frac{3}{2} - y_1}{-x_1}\right) = -\left(\frac{1+2y_1}{6x_1}\right)$$

$9 - 6y_1 = 1 + 2y_1 \quad \Rightarrow y_1 = 1$

$\therefore x_1 = \pm 2$

$\therefore$ Slope of tangent $= \left(\frac{\pm 12}{3}\right)$

$= \pm 4$

$\therefore |n| = 4$

25. If $2x^2 + (a - 10)x + \frac{33}{2} = 2a$, $a \in Z^+$ has real roots, then minimum value of 'a' is equal to

2

Ans. 8

Sol. $D \geq 0$

$(a - 10)^2 - 4(2)\left(\frac{33}{2} - 2a\right) \geq 0$

$(a - 10)^2 - 4(33 - 4a) \geq 0$

$a^2 - 4a - 32 \geq 0 \Rightarrow a \in (-\infty, -4] \cup [8, \infty)$