PART : MATHEMATICS

SECTION – 1

Straight Objective Type

This section contains **21 multiple choice questions.** Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which **Only One** is correct.

1. Let $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{c}$ is nonzero vector and $\vec{b} \times \vec{c} = \vec{b} \times \vec{a}$, $\vec{a}.\vec{c} = 0$ find $\vec{b}.\vec{c}$

(1) $\frac{1}{2}$ (2) $\frac{1}{3}$ (3) $-\frac{1}{2}$ (4) $-\frac{1}{3}$

Ans. **(3)**

Sol. $\vec{a} \times (\vec{b} \times \vec{c}) = \vec{a} \times (\vec{b} \times \vec{a})$

$-(\vec{a}.\vec{b})\,\vec{c} = (\vec{a}.\vec{a})\vec{b} - (\vec{a}.\vec{b})\vec{a}$

$-4\vec{c} = 6(\hat{i} - \hat{j} + \hat{k}) - 4(\hat{i} - 2\hat{j} + \hat{k})$

$-4\vec{c} = 2\hat{i} + 2\hat{j} + 2\hat{k}$

$\vec{c} = -\frac{1}{2}(\hat{i} + \hat{j} + \hat{k})$

$\vec{b}.\vec{c} = -\frac{1}{2}$

2. Let coefficient of x^4 and x^2 in the expansion of $\left(x + \sqrt{x^2 - 1}\right)^6 + \left(x - \sqrt{x^2 - 1}\right)^6$ is α and β then $\alpha - \beta$ is equal to

(1) 48 (2) 60 (3) –132 (4) –60

Ans. **(3)**

$2[{}^6C_0\, x^6 + {}^6C_2\, x^4 (x^2 - 1) + {}^6C_4\, x^2 (x^2 - 1)^2 + {}^6C_6\, (x^2 - 1)^3]$

$= 2[x^6 + 15(x^6 - x^4) + 15x^2(x^4 - 2x^2 + 1) + (-1 + 3x^2 - 3x^4 + x^6)]$

$= 2(32x^6 - 48x^4 + 18x^2 - 1)$

$\alpha = -96$ and $\beta = 36$ $\therefore$ $\alpha - \beta = -132$

3. Differential equation of $x^2 = 4b(y + b)$, where b is a parameter, is

(1) $x\left(\frac{dy}{dx}\right)^2 = 2y\frac{dy}{dx} + x^2$

(2) $x\left(\frac{dy}{dx}\right)^2 = 2y\frac{dy}{dx} + x$

(3) $x\left(\frac{dy}{dx}\right)^2 = y\frac{dy}{dx} + x^2$

(4) $x\left(\frac{dy}{dx}\right)^2 = y\frac{dy}{dx} + 2x^2$

Ans. **(2)**

Sol. $2x = 4by'$ $\Rightarrow$ $b = \frac{x}{2y'}$

So. differential equation is $x^2 = \frac{2x}{y'}.\, y + \left(\frac{x}{y'}\right)^2$

4. Image of (1, 2, 3) w.r.t a plane is $\left(\frac{-7}{3}, \frac{-4}{3}, \frac{-1}{3}\right)$ then which of the following points lie on the plane

(1) (–1, 1,–1) (2) (–1, –1, –1) (3) (–1, –1,1) (4) (1, 1, –1)

Ans. **(4)**

Sol. d.r of normal to the plane

$\frac{10}{3}, \frac{10}{3}, \frac{10}{3}$

1, 1, 1

midpoint of P and Q is $\left(\frac{-2}{3}, \frac{1}{3}, \frac{4}{3}\right)$

equation of plane x + y + z = 1

5. $\lim_{x \to 0} \frac{\int_0^x t\sin(10t)dt}{x}$ is equal to

(1) 1 (2) 10 (3) 5 (4) 0

Ans. **(4)**

Sol. Using L'Hospital

$$\lim_{x\to 0}\frac{x\sin(10x)}{1} = 0$$

6. Let P be the set of points (x, y) such that $x^2 \le y \le -2x + 3$. Then area of region bounded by points in set P is

(1) $\frac{16}{3}$ (2) $\frac{32}{3}$ (3) $\frac{29}{3}$ (4) $\frac{20}{3}$

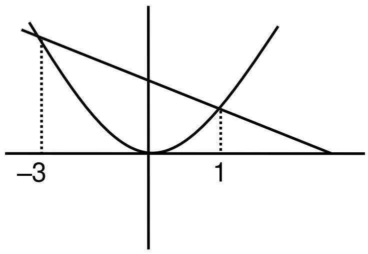
Ans. **(2)**

Sol. Point of intersection of $y = x^2$ & $y = -2x + 3$ is

obtained by $x^2 + 2x - 3 = 0$

$\Rightarrow x = -3, 1$

So, Area $= \int_{-3}^{1}(3-2x-x^2)dx = 3(4) - 2\left(\frac{1^2-3^2}{2}\right) - \left(\frac{1^3+3^3}{3}\right) = 12 + 8 - \frac{28}{3} = \frac{32}{3}$



7. Let $f(x) = \frac{x[x]}{x^2+1}$: (1, 3) → R then range of f(x) is (where [.] denotes greatest integer function)

(1) $\left(0,\frac{1}{2}\right) \cup \left(\frac{3}{5},\frac{7}{5}\right]$ (2) $\left(\frac{2}{5},\frac{1}{2}\right) \cup \left(\frac{3}{5},\frac{4}{5}\right]$

(3) $\left(\frac{2}{5},1\right) \cup \left(1,\frac{4}{5}\right]$ (4) $\left(0,\frac{1}{3}\right) \cup \left(\frac{2}{5},\frac{4}{5}\right]$

Ans. **(2)**

Sol $f(x)\begin{cases}\dfrac{x}{x^2+1}; & x\in(1,2)\\ \dfrac{2x}{x^2+1}; & x\in[2,3)\end{cases}$

$\therefore$ f(x) is a decreasing function

$\therefore$ $\mathbf{y}\in\left(\frac{2}{5},\frac{1}{2}\right)\cup\left(\frac{6}{10},\frac{4}{5}\right]$

$\Rightarrow$ $\mathbf{y}\in\left(\frac{2}{5},\frac{1}{2}\right)\cup\left(\frac{3}{5},\frac{4}{5}\right]$

8. Let A = $\begin{bmatrix}2 & 2\\ 9 & 4\end{bmatrix}$ and I = $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ then value of 10 A^{-1} is –

(1) 4I – A (2) 6 I – A (3) A – 4 I (4) A – 6 I

Ans. **(4)**

Sol. Characteristics equation of matrix 'A' is

$\begin{vmatrix}2-x & 2\\ 9 & 4-x\end{vmatrix}=0$ $\Rightarrow$ $x^2 - 6x - 10 = 0$

$\therefore$ $A^2 - 6A - 10\,I = 0$

$\Rightarrow$ $10A^{-1} = A - 6I$

9. Solution set of $3^x(3^x - 1) + 2 = |3^x - 1| + |3^x - 2|$ contains

(1) singleton set (2) two elements

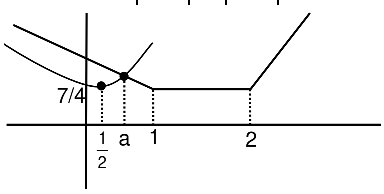
(3) at least four elements (4) infinite elements

Ans. **(1)**

Sol. Let $3^x = t$

$t(t-1) + 2 = |t-1| + |t-2|$

$t^2 - t + 2 = |t-1| + |t-2|$



are positive solution

t = a

$3^x = a$

$x = \log_3 a$ so singleton set

10. Mean and variance of 20 observation are 10 and 4. It was found, that in place of 11, 9 was taken by mistake find correct variance.

(1) 3.99 (2) 3.98 (3) 4.01 (4) 4.02

Ans. **(1)**

Sol. $\frac{\sum x_i}{20} = 10$ ....(i)

$\frac{\sum x_i^2}{20} - 100 = 4$ ....(ii)

$\sum x_i^2 = 104 \times 20 = 2080$

Actual mean $= \frac{200 - 9 + 11}{20} = \frac{202}{20}$

Variance $= \frac{2080 - 81 + 121}{20} - \left(\frac{202}{20}\right)^2$

$= \frac{2120}{20} - (10.1)^2 = 106 - 102.01 = 3.99$

11. $\lambda x + 2y + 2z = 5$

$2\lambda x + 3y + 5z = 8$

$4x + \lambda y + 6z = 10$

for the system of equation check the correct option.

(1) Infinite solutions when $\lambda = 8$ (2) Infinite solutions when $\lambda = 2$

(3) no solutions when $\lambda = 8$ (4) no solutions when $\lambda = 2$

$\lambda x + 2y + 2z = 5$
$2\lambda x + 3y + 5z = 8$
$4x + \lambda y + 6z = 10$

Ans. **(4)**

Sol. $D = \begin{vmatrix} \lambda & 2 & 2 \\ 2\lambda & 3 & 5 \\ 4 & \lambda & 6 \end{vmatrix}$

$D = (\lambda + 8)\ (2 - \lambda)$

for $\lambda = 2$

$D_1 = \begin{vmatrix} 5 & 2 & 2 \\ 8 & 3 & 5 \\ 10 & 2 & 6 \end{vmatrix}$

$= 5[18 - 10] - 2\ [48 - 50] + 2\ (16 - 30]$

$= 40 + 4 - 28 \neq 0$

No solutions for $\lambda = 2$

12. For an A.P. $T_{10} = \frac{1}{20}$; $T_{20} = \frac{1}{10}$ Find sum of first 200 term.

(1) $201\ \frac{1}{2}$ (2) $101\ \frac{1}{2}$ (3) $301\ \frac{1}{2}$ (4) $100\ \frac{1}{2}$

Ans. **(4)**

Sol. $T_{10} = \frac{1}{20} = a + 9d$ …………..(i)

$T_{20} = \frac{1}{10} = a + 19d$ …………..(ii)

$\Rightarrow \quad a = \frac{1}{200},\ d = \frac{1}{200} \quad \Rightarrow \quad S_{200} = \frac{200}{2}\left[\frac{2}{200} + \frac{199}{200}\right] = \frac{201}{2} = 100\ \frac{1}{2}$

13. Let $\alpha = \frac{-1 + i\sqrt{3}}{2}$ and $a = (1+\alpha)\sum_{k=0}^{100}\alpha^{2k}$, $b = \sum_{k=0}^{100}\alpha^{3k}$. If a and b are roots of quadratic equation then quadratic equation is

(1) $x^2 - 102x + 101 = 0$ (2) $x^2 - 101x + 100 = 0$
(3) $x^2 + 101x + 100 = 0$ (4) $x^2 + 102x + 100 = 0$

Ans. **(1)**

Sol. $\alpha = \omega$, $b = 1 + \omega^3 + \omega^6 + \ldots\ldots = 101$

$a = (1 + \omega)(1 + \omega^2 + \omega^4 + \ldots\ldots\omega^{198} + \omega^{200})$

$$= (1+\omega)\frac{\left(1-(\omega^2)^{101}\right)}{1-\omega^2} = \frac{(1+\omega)(1-\omega)}{1-\omega^2} = 1$$

Equation : $x^2 - (101 + 1)x + (101) \times 1 = 0$ $\Rightarrow x^2 - 102x + 101 = 0$

14. Let f(x) is a three degree polynomial for which $f'(-1) = 0$, $f''(1) = 0$, $f(-1) = 10$, $f(1) = 6$ then local minima of f(x) exist at

(1) $x = 3$ (2) $x = 2$ (3) $x = 1$ (4) $x = -1$

Ans. (1)

Sol. Let $f(x) = ax^3 + bx^2 + cx + d$

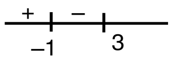
$a = \frac{1}{4}$ $d = \frac{35}{4}$

$b = \frac{-3}{4}$ $c = -\frac{9}{4}$

$\Rightarrow f(x) = a(x^3 - 3x^2 - 9x) + d$

$f'(x) = \frac{3}{4}(x^2 - 2x - 3)$

$\Rightarrow f'(x) = 0 \Rightarrow x = 3, -1$



local minima exist at $x = 3$ $x = 3$ पर निम्निष्ठ मान है

15. Let A and B are two events such that P(exactly one) = $\frac{2}{5}$, $P(A \cup B) = \frac{1}{2}$ then $P(A \cap B) =$

(1) $\frac{1}{10}$ (2) $\frac{2}{9}$ (3) $\frac{1}{8}$ (4) $\frac{1}{12}$

Ans. (1)

Sol. P(exactly one) $= \frac{2}{5}$

$\Rightarrow P(A) + P(B) - 2P(A \cap B) = \frac{2}{5}$

$P(A \cup B) = \frac{1}{2}$

$\Rightarrow P(A) + P(B) - P(A \cap B) = \frac{1}{2}$

$\therefore P(A \cap B) = \frac{1}{2} - \frac{2}{5} = \frac{5-4}{10} = \frac{1}{10}$

16. Let $I = \int_1^2 \frac{1dx}{\sqrt{2x^3 - 9x^2 + 12x + 4}}$ then

(1) $\frac{1}{9} < I^2 < \frac{1}{8}$

(2) $\frac{1}{3} < I^2 < \frac{1}{2}$

(3) $\frac{1}{9} < I < \frac{1}{8}$

(4) $\frac{1}{3} < I < \frac{1}{2}$

Ans. **(1)**

Sol. $f(x) = \frac{1}{\sqrt{2x^3 - 9x^2 + 12x + 4}}$

$f'(x) = \frac{-1}{2} \frac{(6x^2 - 18x + 12)}{(2x^3 - 9x^2 + 12x + 4)^{\frac{3}{2}}}$

$= \frac{-6(x-1)(x-2)}{2(2x^3 - 9x^2 + 12x + 4)^{\frac{3}{2}}}$

$f(1) = \frac{1}{3}$, $f(2) = \frac{1}{\sqrt{8}}$

$\frac{1}{3} < I < \frac{1}{\sqrt{8}}$

17. Normal at (2, 2) to curve $x^2 + 2xy - 3y^2 = 0$ is L. Then perpendicular distance from origin to line L is

(1) $4\sqrt{2}$ (2) 2 (3) $2\sqrt{2}$ (4) 4

Ans. **(3)**

Sol. $x^2 + 2xy - 3y^2 = 0$

$x^2 + 3xy - xy - 3y^2 = 0$

$(x - y)(x + 3y) = 0$

$x - y = 0$ $x + 3y = 0$

(2, 2) satisfy $x - y = 0$

Normal

$x + y = \lambda$

$\lambda = 4$

Hence $x + y = 4$

perpendicular distance from origin = $\left|\frac{0 + 0 - 4}{\sqrt{2}}\right| = 2\sqrt{2}$

18. Which of the following is tautology-

(1) $\sim(p \vee \sim q) \rightarrow (p \vee q)$ (2) $(\sim p \vee q) \rightarrow (p \vee q)$

(3) $\sim(p \wedge \sim q) \rightarrow (p \vee q)$ (4) $\sim(p \vee \sim q) \rightarrow (p \wedge q)$

Ans. (1)

Sol. $(\sim p \wedge q) \rightarrow (p \vee q)$

$\sim\{(\sim p \wedge q) \wedge (\sim p \wedge \sim q)\}$

$\sim\{\sim p \wedge f\}$

19. If a hyperbola has vertices (±6, 0) and P(10, 16) lies on it, then the equation of normal at P is

(1) $2x + 5y = 100$ (2) $2x + 5y = 10$ (3) $2x - 5y = 100$ (4) $5x + 2y = 100$

Ans. (1)

Sol. Vertex is at (±6, 0)

$\therefore$ a = 6

Let the hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Putting point P(10, 16) on the hyperbola

$\frac{100}{36} - \frac{256}{b^2} = 1 \quad \Rightarrow \quad b^2 = 144$

$\therefore$ hyperbola is $\frac{x^2}{36} - \frac{y^2}{144} = 1$

$\therefore$ equation of normal is $\frac{a^2x}{x_1} + \frac{b^2y}{y_1} = a^2 + b^2$

$\therefore$ putting we get $2x + 5y = 100$

20. If y = mx + c is a tangent to the circle $(x-3)^2 + y^2 = 1$ and also the perpendicular to the tangent to the circle $x^2 + y^2 = 1$ at $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$, then

(1) $c^2 + 6c + 7 = 0$ (2) $c^2 - 6c + 7 = 0$ (3) $c^2 + 6c - 7 = 0$ (4) $c^2 - 6c - 7 = 0$

Ans. (1)

Sol. Slope of tangent to $x^2 + y^2 = 1$ at $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

$x^2 + y^2 = 1$

$2x + 2yy' = 0$

$y' = -\frac{x}{y} = -1$

y = mx + c is tangent of $x^2 + y^2 = 1$

so m = 1

y = x + c

now distance of (3, 0) from y = x + c is

$\left|\frac{c+3}{\sqrt{2}}\right| = 1$

$c^2 + 6c + 9 = 2$

$c^2 + 6c + 7 = 0$

21. Let $\frac{\sqrt{2}\sin\alpha}{\sqrt{1+\cos 2\alpha}} = \frac{1}{7}$ and $\sqrt{\frac{1-\cos 2\beta}{2}} = \frac{1}{\sqrt{10}}$ where $\alpha, \beta \in \left(0, \frac{\pi}{2}\right)$. Then $\tan(\alpha + 2\beta)$ is equal to

Ans. (1)

Sol. $\frac{\sqrt{2}\sin\alpha}{\sqrt{2}\cos\alpha} = \frac{1}{7}$ and $\frac{\sqrt{2}\sin\beta}{\sqrt{2}} = \frac{1}{\sqrt{10}}$

$\tan\alpha = \frac{1}{7}$

$\sin\beta = \frac{1}{\sqrt{10}}$

$\tan\beta = \frac{1}{3}$

$$\tan 2\beta = \frac{2.\frac{1}{3}}{1-\frac{1}{9}} = \frac{\frac{2}{3}}{\frac{8}{9}} = \frac{3}{4}$$

$$\tan(\alpha + 2\beta) = \frac{\tan\alpha + \tan 2\beta}{1-\tan\alpha\tan 2\beta} = \frac{\frac{1}{7}+\frac{3}{4}}{1-\frac{1}{7}.\frac{3}{4}} = \frac{\frac{4+21}{28}}{\frac{25}{28}} = 1$$

SECTION – 2

- This section contains **FIVE (03)** questions. The answer to each question is **NUMERICAL VALUE** with two digit integer and decimal upto one digit.
- If the numerical value has more than two decimal places **truncate**/**round-off** the value upto **TWO** decimal places.
  - Full Marks : **+4** If ONLY the correct option is chosen.
  - Zero Marks : **0** In all other cases

22. The number of four letter words that can be made from the letters of word "EXAMINATION" is

Ans. 2454

Sol. EXAMINATION

2N, 2A, 2I, E, X, M, T, O

Case I All are different so $^8P_4 = \frac{8!}{4!} = 8.7.6.5 = 1680$

Case II 2 same and 2 different so $^3C_1 . ^7C_2 . \frac{4!}{2!} = 3.21.12 = 756$

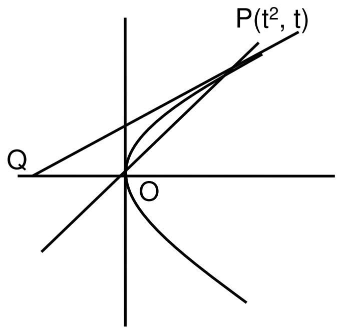
Case III 2 same and 2 same so $^3C_2 . \frac{4!}{2!.2!} = 3.6 = 18$

$\therefore$ Total = 1680 + 756 + 18 = 2454

23. Let the line y = mx intersects the curve $y^2 = x$ at P and tangent to $y^2 = x$ at P intersects x-axis at Q. If area (ΔOPQ) = 4, find m (m > 0)

Ans. 0.5

Sol.



$2ty = x + t^2$

$Q(-t^2, 0)$

$$\frac{1}{2}\left|\begin{vmatrix} 0 & 0 & 1 \\ t^2 & t & 1 \\ -t^2 & 0 & 1 \end{vmatrix}\right| = 4$$

$|t|^3 = 8$

$t = \pm 2 \quad (t > 0)$

$m = \frac{1}{2}$

24. $\sum_{n=1}^{7} \frac{n(n+1)(2n+1)}{4}$ is equal to

Ans. 504

Sol. $\frac{1}{4}\left[\sum_{n=1}^{7}\left(2n^3 + 3n^2 + n\right)\right]$

$$\frac{1}{4}\left[2\left(\frac{7.8}{2}\right)^2 + 3\left(\frac{7.8.15}{6}\right) + \frac{7.8}{2}\right]$$

$\frac{1}{4}[2 \times 49 \times 16 + 28 \times 15 + 28]$

$\frac{1}{4}[1568 + 420 + 28] = 504$