JEE Main (Phase-II) 2020

Memory Based Questions & Solutions

SUBJECT

MATHEMATICS

Date: 05 September, 2020 (Shift-1)

Time: 9 AM to 12 PM

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1. The number of four letters word while each consisting 2 distinct and two alike letters taken from word 'SYLLABUS'

(1) 240 (2) 144 (3) 288 (4) 432

Ans. (1)

Sol. SYLLABUS

S,2, L-2, A, B, Y, U

Required $= {}^2C_1 . {}^5C_2 . \frac{4!}{2!} = 2.10.\frac{24}{2} = 240$

2. The common tangent of curves $x^2 = 4y$ and $y^2 = 4x$ also touches the curve $x^2 + y^2 = c^2$ then find value of c^2

(1) $\frac{1}{2}$ (2) $\frac{1}{4}$ (3) $\frac{1}{\sqrt{2}}$ (4) 1

Ans. (1)

Sol. $y^2 = 4x$ & $x^2 = 4y$

any tangent of $y^2 = 4x$ is $y = mx + \frac{1}{m}$

it also tangent for $x^2 = 4y$

$\therefore \frac{1}{m} = -m^2 \Rightarrow m = -1$

$\therefore$ common tangent is $y = = -x - 1$, it also touches $x^2 + y^2 = c^2$

$\therefore 1 = c^2 .(1+1) \Rightarrow c^2 = \frac{1}{2}$

3. If P is point lying on $\frac{x^2}{16} + \frac{y^2}{9} = 1$ and $A(\sqrt{7}, 0), B(-\sqrt{7}, 0)$ are two points then $PA + PB = ?$

(1) 4 (2) 10 (3) 8 (4) 5

Ans. (3)

Sol. For ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$, $a = 4$, $b = 3$, $e = \sqrt{1 - \frac{9}{16}} = \sqrt{\frac{7}{4}}$

A and B are foci

than PA+ PB = 2a =2 (4) =8

4. If distance between lines $2x - y + 3 = 0$ and $4x - 2y + \alpha = 0$ is $\frac{1}{\sqrt{5}}$ while distance between $2x - y + 3 = 0$ and $6x - 3y + \beta = 0$ is $\frac{2}{\sqrt{5}}$ then $\alpha + \beta$ can be?

(1) 23 (2) 10 (3) 20 (4) 12

Ans. (1)

Sol. $2x - y + 3 = 0$ ......(i)

$4x - 2y + \alpha = 0 \Rightarrow 2x - y + \frac{\alpha}{2} = 0$........(ii)

$6x - 3y + \beta = 0 \Rightarrow 2x - y + \frac{\beta}{3} = 0$..........(iii)

$$d_1 = \frac{\left|\frac{\alpha}{2} - 3\right|}{\sqrt{2^2 + 1^2}} = \frac{1}{\sqrt{5}} \Rightarrow |\alpha - 6| = 2 \Rightarrow \alpha - 6 = 2, -2 \Rightarrow \alpha = 8, 4$$

$$d_2 = \frac{\left|\frac{\beta}{3} - 3\right|}{\sqrt{2^2 + 1^2}} = \frac{2}{\sqrt{5}} \Rightarrow |\beta - 9| = 6 \Rightarrow \beta - 9 = 6, -6 \Rightarrow \beta = 15, 3$$

$\alpha + \beta = 8 + 15$ or $8 + 3$ or $4 + 15$ or $4 + 3 = 23$ or 11 or 19 or 7

5. $p \Leftrightarrow \sim q$ is equivalent to

(1) $(p \wedge \sim q) \wedge (p \vee q)$ (2) $(\sim p \vee q) \wedge (q \vee p)$ (3) $(p \vee q) \wedge (q \vee p)$ (4) $p \Leftrightarrow q$

Ans. (2)

Sol. $p \Leftrightarrow \sim q \equiv (\sim p \vee q) \wedge (q \vee p)$

6. From a survey, 73% like coffee, 65% like tea, and 55% like both coffee and tea then how many person do not like both tea and coffee

(1) 16% (2) 17% (3) 18% (4) 20%

Ans. (2)

Sol. $n(C) = 73, \quad n(T) = 65, \quad n(C \cap T) = 55$

$n(C \cup T) = 73 + 65 - 55 = 83$

$\therefore n(C' \cup T) = n(\cup) - n(C \cup T)$

$= 100 - 83$

$= 17$

7. Sum of series $\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{13} + \ldots$ (10 term), is equal to

(1) $\left(\tan^{-1}11 - \frac{\pi}{4}\right)$ (2) $\left(\tan^{-1}11 + \frac{\pi}{4}\right)$ (3) $\left(\tan^{-1}12 + \frac{\pi}{4}\right)$ (4) $\left(\tan^{-1}10 - \frac{\pi}{4}\right)$

Ans. (1)

Sol. $S = \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{13} + \ldots\ldots\text{upto } 10\text{term}$

$S = \tan^{-1}\left(\frac{2-1}{1+1.2}\right) + \tan^{-1}\left(\frac{3-2}{1+2.3}\right) + \tan^{-1}\left(\frac{4-3}{1+3.4}\right) + \ldots\ldots\ldots + \tan^{-1}\left(\frac{11-10}{1+11.10}\right)$

$S = (\tan^{-1}2 - \tan^{-1}1) + (\tan^{-1}3 - \tan^{-1}2) + \ldots\ldots + (\tan^{-1}11 - \tan^{-1}10)$

$S = \tan^{-1}11 - \tan^{-1}1$

$S = \tan^{-1}(11) - \frac{\pi}{4}$

8. If $\frac{dy}{2+y} = \frac{e^x dx}{5+e^x}$, where $y(0) = 4$, then find $y(\log_e 13)$

Ans. (16)

Sol. Given $\frac{dy}{2+y} = \frac{e^x dx}{5+e^x}$

$\ell n(2+y) = \ell n(5+e^x) + \ell nC$

$y = (5 + e^x)C - 2$

$y(0) = 4 \therefore \quad 4 = 6C - 2 \Rightarrow \quad C = 1$

$\therefore \quad y = 3 + e^x$

$\therefore \quad y = (\log_e 13) = 3 + e^{\log_e 13} = 3 + 13 = 16$

9. Mean and variance of 7 observation 2, 4, 10, 12, 14, x, y are 8 and 18 respectively find xy=

Ans. (48)

Sol. $\bar{x} = \dfrac{2+4+10+12+14+x+y}{7} = 8 \Rightarrow 42 + x + y = 56 \Rightarrow x + y = 14$

$$\sigma^2 = \frac{\Sigma x_i^2}{n} = \left(\frac{\Sigma x_i}{n}\right)^2$$

$$16 = \frac{4+16+100+144+196+x^2+y^2}{7} - (8)^2$$

$$\Rightarrow 16 + 64 = \frac{460 + x^2 + y^2}{7}$$

$$\Rightarrow 560 = 460 + x^2 + y^2 \Rightarrow x^2 + y^2 = 100.......(2) \Rightarrow xy = 48$$

10. If $P(x) = x^2 - x - 2$ and α is positive root of $P(x) = 0$ then $\lim\limits_{x\to\alpha^+} \dfrac{\sqrt{1-\cos P(x)}}{x-\alpha}$ is equal to

(1) $\dfrac{\sqrt{3}}{2}$ (2) $\dfrac{3}{\sqrt{2}}$ (3) $\sqrt{\dfrac{3}{2}}$ (4) $\dfrac{1}{\sqrt{2}}$

Ans. (2)

Sol. $P(x) = 0$

$x^2 - x - 2 = 0$

$(x - 2)(x + 1) = 0$

$x = 2, -1 \quad \therefore \quad \alpha = 2$

Now $\lim\limits_{x\to 2^+} \dfrac{\sqrt{1-\cos(x^2-x-2)}}{x-2} \Rightarrow \lim\limits_{x\to 2^+} \dfrac{\sqrt{2\sin^2\left(\dfrac{x^2-x-2}{2}\right)}}{x-2}$

$$\Rightarrow \lim_{x\to 2^+} 2\frac{\left|\sin\left(\dfrac{x^2-x-2}{2}\right)\right|}{x-2} \Rightarrow \text{for } x \to 2^+, \frac{x^2-x-2}{2} \to 0^+$$

$$\lim_{x\to 2^+} \frac{\sqrt{2}\sin\dfrac{(x^2-x-2)}{2}}{x-2} \cdot \frac{\dfrac{x^2-x-2}{2}}{(x-2)} \Rightarrow \lim_{x\to 2^+} \frac{1}{\sqrt{2}} \cdot \frac{(x-2)(x+1)}{(x-2)} = \frac{3}{\sqrt{2}}$$

11. If $2^{10} + 2^9 . 3 + 2^8 . 3^2 + ... + 3^{10} = S . 2^{11}$ then S=

(1) $\dfrac{3^{11}}{2^{11}}$ (2) $\dfrac{3^{11}}{2^{11}} - 1$ (3) $\dfrac{2^{11}}{3^{11}} - 1$ (4) $\dfrac{3^{11}}{2^{11}} - 2$

Ans. (4)

Sol. $S' = 2^{10} + 2^9.3 + 2^8 3^2 + ... + 2.3^9 + 3^{10}$

G.P. $\rightarrow a = 2^{10}, r = \frac{3}{2}, n = 11$

$$S' = 2^{10}.\frac{\left(\left(\frac{3}{2}\right)^{11} - 1\right)}{\frac{3}{2} - 1} = 2^{11}\left(\frac{3^{11}}{2^{11}} - 1\right)$$

12. Four different dice are thrown independently 27 times, then find the expectation of number of times if at least two of them shows either 3 or 5.

Ans. (11)

Sol. P(at least 2 show 3 or 5) $= {}^4C_2.\left(\frac{2}{6}\right)^2\left(\frac{4}{6}\right)^2 + {}^4C_3\left(\frac{2}{6}\right)^3\left(\frac{4}{6}\right) + {}^4C_4\left(\frac{2}{6}\right)^4$

$= \frac{384 + 128 + 16}{6^4} = \frac{11}{27}$

n = 27

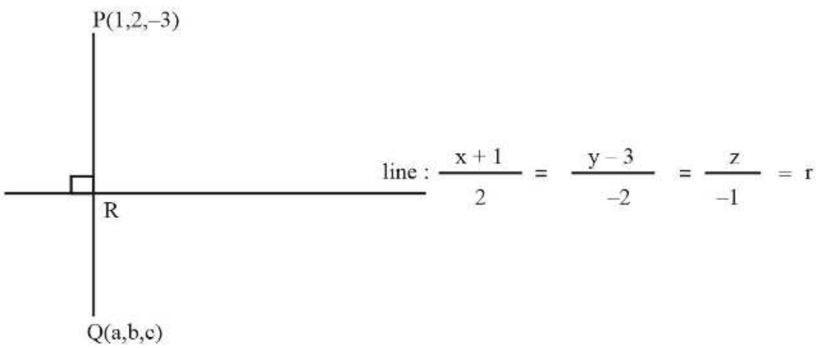
$\therefore$ expectation of number of times = np

$= 27.\frac{11}{27} = 11$

13. If (a, b, c) is the image of the point (1, 2, –3) in the line $\frac{x+1}{2} = \frac{y-3}{-2} = \frac{z}{-1}$, then value of a + b + c is equal to:

Ans. (2)

Sol.



R(– 1 + 2r, 3 – 2r, –r)

dr's of PR are (2 – 2r, – 1 + 2r, – 3 + r)

Then 2(2 – 2r) + 2(1 – 2r) + 1(3 – r) = 0

9 – 9r = 0 $\Rightarrow$ r = 1

R(1, 1, – 1)

then a+ 1 = 2   b + 2 = 2   c – 3 = – 2

a = 1   b = 0   c = 1

$\therefore$ a + b + c = 2

14. $\int\left(e^{2x}+2e^{x}-e^{-x}-1\right)\left(e^{e^{x}+e^{-x}}\right)dx=g(x)e^{\left(e^{x}+e^{-4}\right)}+C$ then value of g(x) is equal to

(1) 1 (2) 2 (3) 3 (4) 4

Ans. (2)

Sol. $I=\int(e^{2x}+2e^{x}-e^{-x}-1)e^{(e^{x}+e^{-x})}dx$

$I=\int(e^{2x}+e^{x}-1)e^{(e^{x}+e^{-x})}dx+\int\left(e^{x}-e^{-x}\right)e^{e^{x}+e^{-x}}dx$

$I=\int(e^{x}+1-e^{-x})e^{e^{x}+e^{-x}+x}dx+e^{e^{x}+e^{-x}}$

$e^{x}+e^{-x}+x=du$

$\left(e^{x}-e^{-x}+1\right)dx=du$

$I=e^{e^{x}+e^{-x}+x}+e^{e^{x}+e^{-x}}=e^{e^{x}+e^{-x}}(e^{x}+1)$

then $g(x)=e^{x}+1$

$g(0)=2$

15. If function $f(x)=\begin{cases}k_1(x-\pi)^2-1 & x\le\pi\\ k_2\cos x & x>\pi\end{cases}$

os twice differentiable function. Then ordered pair (k_1, k_2) is:

(1) $\left(-\frac{1}{2},1\right)$ (2) $\left(\frac{1}{2},1\right)$ (3) $\left(\frac{1}{2},-1\right)$ (4) $\left(-\frac{1}{2},-1\right)$

Ans. (2)

Sol. f(c) is differentiable then will also continuous

then $f(\pi)=-1$, $f(\pi^+)=-k_2$

$k_2=1$

Now

$f'(x)=\begin{cases}2k_1(x-\pi) & x\le\pi\\ -k_2\sin x & x>\pi\end{cases}$ then $f'(\pi^-)=f'(\pi^+)=0$

$f'(x)=\begin{cases}2k_1 & x\le\pi\\ -k_2\cos x & x>\pi\end{cases}$

then $2k_1=k_2$

$k_1=\frac{1}{2}$

16. If volume of parallelopiped whose coterminous edges are $\vec{a}=\hat{i}+\hat{j}+n\hat{k},\vec{b}=2\hat{i}+4\hat{j}-n\hat{k},\vec{c}=\hat{i}+n\hat{j}+3\hat{k}$ is 158 cube units. Then

(1) n=7 (2) n =9 (3) $\overline{a}.\overline{c}=17$ (4) $\overline{b}.\overline{c}=10$

Ans. (4)

Sol. Volume of parallelepiped $v\,|[\vec{a}\vec{b}\vec{c}]|$

$$v=\begin{vmatrix}1 & 1 & n\\ 2 & 4-n & \\ 1 & n & 3\end{vmatrix}=\pm158$$

$1(12 + n^2) - 1(6 + n) + n(2n - 4) = \pm 158$

$3n^2 - 5n - 152 = 0$ or $3n^2 - 5n + 164 = 0$ D < 0 (no real roots)

$n = 8, -\frac{19}{3} \Rightarrow n = 8$

then $\vec{b}.\vec{c} = 2 + 4n - 3n = 10$

$\vec{a}\,\vec{c} = 1 + n + 3n = 33$

17. If coefficient of x in the expansion of $\left(x^m + \frac{1}{x^2}\right)^{22}$ is 1540 and $m \in N$ then value of m is

Ans. (13)

Sol. $T_{r+1} = {}^{22}C_r (x^m)^{22-r} x^{-2r}$

$T_{r+1} = {}^{22}C_r x^{m(22-r)-2r}$

$22m - mr - 2r = 1$

$22m - 1 = r(m + 2)$

$r = \frac{22m-1}{m+2}$

$r = \frac{22m+44-45}{m+2}$

$r = 22 - \frac{3.3.5}{m+2}$

so possible value of m = 1, 3, 7, 13, 43

but ${}^{20}C_r = 1540$

only possible condition is m = 13

18. If the point P lies on the curve $4x^2 + 5y^2 - 20 = 0$ is farthest from the point Q(0, –4) then $(PQ)^2$ is equal to

Ans. (36)

Sol. Equation $\frac{x^2}{5} + \frac{y^2}{4} = 1$ then $P\left(\sqrt{5}\cos\theta, 2\sin\theta\right)$

$(PQ)^2 = 5\cos^2\theta + 4(\sin\theta + 2)^2 = \cos^2\theta + 16\sin\theta + 20 = -\sin^2\theta + 16\sin\theta + 21$

$= 85 - (\sin\theta - 8)^2$

$(PQ)^2_{max} = 85 - 49 = 36, \because (\sin\theta - 8)^2 \in [49, 81]$

19. If $3^{2\sin 2\theta - 1}$, 14 and $3^{4 - 2\sin 2\theta}$ are first three terms of an AP for some θ. then 6th term of A.P. is

Ans. (66)

Sol. a, b, c are in AP then

$2b = a + c$

$28 = 3^{2\sin 2\theta - 1} + 3^{4 - 2\sin 2\theta}$

Put $3^{2\sin 2\theta} = x$

$28 = \frac{x}{3} + \frac{81}{x} \Rightarrow x^2 - 84x + 243 = 0$ $(x - 3)(x - 81) = 0$

$3^{2\sin 2\theta} = 3$ or 3^4 $2\sin 2\theta = 1$ or 4

$\sin 2\theta = \frac{1}{2}$

terms are 1, 14, 27,............ then $T_6 = 1 + 5(13)$

20. The product of roots of the equation $9x^2 - 18|x| + 5 = 0$ is equal to

(1) $\frac{25}{81}$ (2) $\frac{81}{25}$ (3) $\frac{29}{81}$ (4) 0

Ans. (1)

Sol. $\therefore\ x^2 = |x|^2 = t$ let

$69^2 - 18t + 5 = 0$

$(3t - 1)(3t - 5) = 0$

$|x| = \frac{1}{3}, \frac{5}{3}$

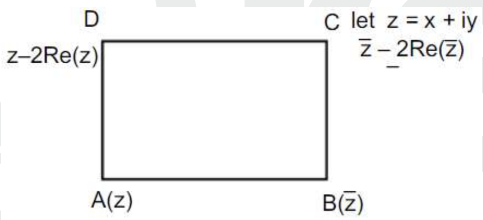
product of roots $= \frac{1}{3}\left(-\frac{1}{3}\right)\left(\frac{5}{3}\right)\left(\frac{-5}{3}\right) = \frac{25}{81}$

21. If four complex number $z, \overline{z}, \overline{z} - 2\,\text{Re}(\overline{z})$ and $z - 2\,\text{Re}(z)$ reresent the vertices of a square of side length 4 units, in argand plane, then the value of $|z|$ is equal to

(1) $\sqrt{2}$ (2) $2\sqrt{2}$ (3) 2 (4) 4

Ans. (2)

Sol.



$\therefore$ length of side = 4

then $|z - \overline{z}| = 4$

$|2iy| = 4$

$|y| = 2$

also $|z - (z - 2\text{Re}(z)| = 4$

$|2x| = 4 \Rightarrow |x| = 2$

$|z| = \sqrt{x^2 + y^2} = 2\sqrt{2}$

22. The value of integration $I = \int_{-\pi/2}^{\pi/2} \frac{1}{1 + e^{\sin x}} dx$ is

(1) $-\frac{\pi}{2}$ (2) π (3) $\frac{\pi}{2}$ (4) $-\pi$

Ans. (3)

Sol. $I = \int_{-\pi/2}^{\pi/2} \frac{1}{1 + e^{\sin x}} dx. I = \int_{-\pi/2}^{\pi/2} \frac{1}{1 + e^{\sin x}} dx$ $\quad I = \int_{-\pi/2}^{\pi/2} \frac{e^{\sin x}}{1 + e^{\sin x}} dx. \begin{cases} \text{Replace} \\ x \to (a + b - x) \end{cases}$

$\int_a^b (f(x)dx = \int_c^b f(a + b - x)dx$ $\quad 2I = \int_{-\pi/2}^{\pi/2} 1dx \Rightarrow I = \frac{1}{2}\int_{-\pi/2}^{\pi/2} dx$

$I = \frac{1}{2}[x]_{-\pi/2}^{\pi/2} \Rightarrow 1 = \frac{\pi}{2}$

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JEE MAIN 2020
PHASE 1

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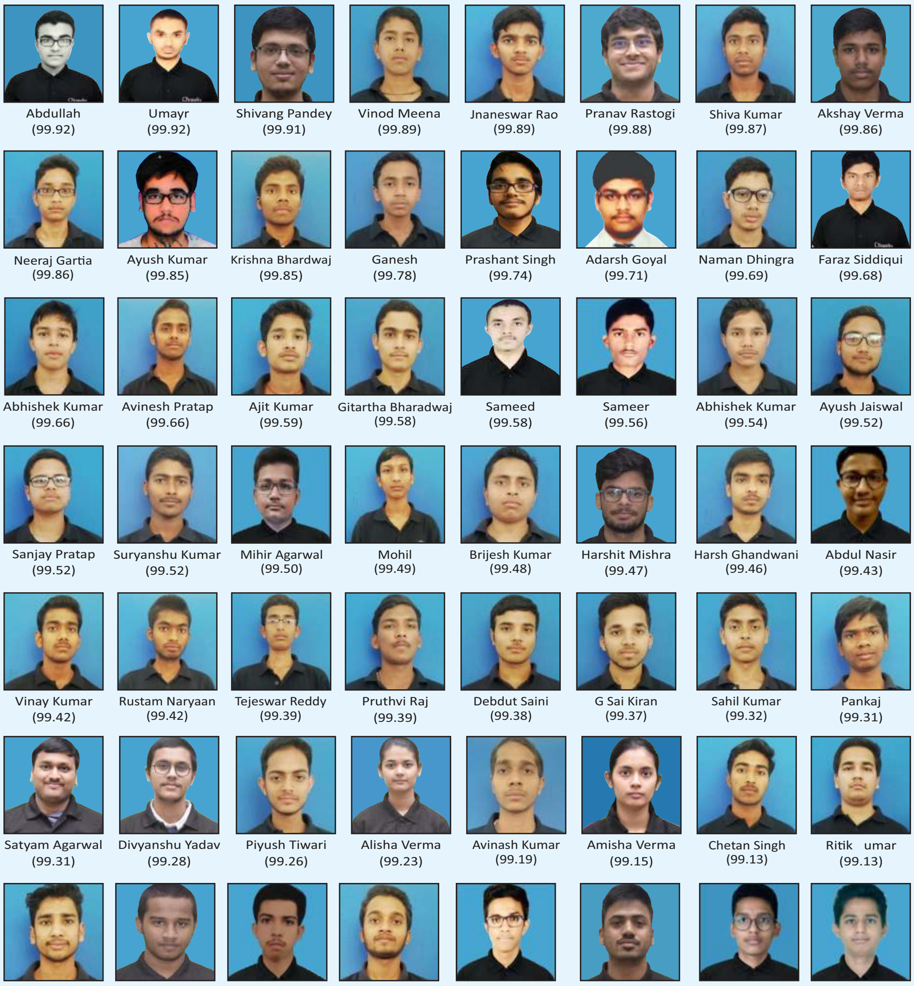
65 Students Above 99 Percentile

145 Students Above 98 Percentile

208 Students Above 97 Percentile



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Neeraj Gartia (99.86)	Ayush Kumar (99.85)	Krishna Bhardwaj (99.85)	Ganesh (99.78)	Prashant Singh (99.74)	Adarsh Goyal (99.71)	Naman Dhingra (99.69)	Faraz Siddiqui (99.68)
Abhishek Kumar (99.66)	Avinesh Pratap (99.66)	Ajit Kumar (99.59)	Gitartha Bharadwaj (99.58)	Sameed (99.58)	Sameer (99.56)	Abhishek Kumar (99.54)	Ayush Jaiswal (99.52)
Sanjay Pratap (99.52)	Suryanshu Kumar (99.52)	Mihir Agarwal (99.50)	Mohil (99.49)	Brijesh Kumar (99.48)	Harshit Mishra (99.47)	Harsh Ghandwani (99.46)	Abdul Nasir (99.43)
Vinay Kumar (99.42)	Rustam Naryaan (99.42)	Tejeswar Reddy (99.39)	Pruthvi Raj (99.39)	Debdut Saini (99.38)	G Sai Kiran (99.37)	Sahil Kumar (99.32)	Pankaj (99.31)
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SCHOOL INTEGRATED PROGRAM (SIP)

Tradition of Gravity Continues, Once Again Historical Result, 100% Students Cracked JEE Main (Based on Last Yr Cut off)

2020
80 Out of 80 Cracked JEE Main

We had three Batches of 55, 15 and 10.

Many Top Ranks are from these Batches

2019
79 Out of 80 in JEE Main	50 Out of 79 in JEE Adv.

2018
83 Out of 85 in JEE Main	62 Out of 83 in JEE Adv.

2017
80 Out of 85 in JEE Main	63 Out of 80 in JEE Adv.

2016
39 Out of 40 in JEE Main	31 Out of 39 in JEE Adv.

Selections Engineering 2019

gravity
Orienting Intelligence


Tarun
194 AIR (General)


Aniket Agarwal
337 AIR (General)


Shubh Sahu
494 AIR (General)


Shlok Nemani
497 AIR (General)

50 out of 79 Cracked JEE Advanced from SIP (School Integrated Program)

4 Ranks under 500 (General Category) | 2 Ranks under 10 (Reserved Category)

126 Selections in JEE Advanced | 61 Students above 99 Percentile in JEE Main 2019


Sanjana
AIR - 3*


Akash
AIR - 4*


Priyanka
AIR - 68*

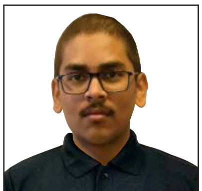

Bibek Lakra
AIR - 150*


Neha Raj
AIR - 177*


Arindam
AIR - 809 (General EWS)


Priyam
AIR - 1378 (General)


Mihir Chawla
AIR - 2237 (General)


Madhur Kumar
AIR - 2382 (General)


Manish Kumar
AIR - 2388 (General)


Saumya Raj
AIR - 2656 (General)


Raghav
AIR - 2659 (General)


Ritveek
AIR - 2709 (General)


Vanshaj
AIR - 2787 (General)


Subir Gupta
AIR - 2881 (General)


Aryan Rastogi
AIR - 3167 (General)


Devansh
AIR - 3600 (General)


Abhisht Bose
AIR - 3784 (General)