JEE Main (Phase-II) 2020

Memory Based Questions & Solutions

SUBJECT

MATHEMATICS

Date: 05 September, 2020 (Shift-2)

Time: 3 PM to 6 PM

HAZRATGANJ
9A, Opp. St. Francis College,
Shahnajaf Road, Hazratganj,
Lucknow -
Call : 0522-4242040, 7518804005

INDIRA NAGAR
D-3221, Sector D
Near Munshipulia,
Lucknow
Call : 0522-4954072, 7518804004

GOMTI NAGAR
CP-15, 16, II Floor, SS Tower,
Near Sahara Hospital,
Viraj Khand-4, Lucknow
Call : 0522-2986600, 9369845766

ALIGANJ
A-1/4, II Floor, Above Bank of Baroda ,
Sector-A, Kapoorthala,
Lucknow -
Call : 7518804003

www.gravityclasses.org | enquiry@gravityclasses.org | www.gravityclasses.live

1. If α, β are roots of equation $7x^2 - 3x + 2 = 0$ then find the value of $\frac{\alpha}{1-\alpha^2} + \frac{\beta}{1-\beta^2}$

(1) $\frac{7}{24}$ (2) $\frac{5}{24}$ (3) $\frac{24}{5}$ (4) $\frac{24}{7}$

Ans. (2)

Sol. $\alpha + \beta = \frac{3}{7} . \alpha\beta = \frac{2}{7}$

$$\frac{\alpha}{1-\alpha^2} + \frac{\beta}{1-\beta^2} = \frac{(\alpha+\beta) - \alpha\beta(\alpha+\beta)}{(1-\alpha^2)(1-\beta^2)} = \frac{(\alpha+\beta) - \alpha\beta(\alpha+\beta)}{1+\alpha(\alpha\beta)^2 - (\alpha^2+\beta^2)}$$

$$\Rightarrow \frac{(\alpha+\beta) - \alpha\beta(\alpha+\beta)}{1+\alpha(\alpha\beta)^2 - (\alpha^2+\beta^2) + 2\alpha\beta} = \frac{\frac{3}{7} - \frac{2}{7}\left(\frac{3}{7}\right)}{1+\left(\frac{2}{7}\right)^2 - \left(\frac{3}{7}\right)^2 + 2\left(\frac{2}{7}\right)} \Rightarrow \frac{\left(\frac{15}{49}\right)}{\left(\frac{72}{49}\right)} = \frac{15}{72} = \frac{5}{24}$$

2. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that $|\vec{a}| = 2, |\vec{b}| = 4, |\vec{c}| = 0, \vec{b}.\vec{c} = 0, \vec{b}.\hat{a} = \vec{c}.\hat{a}$ then find the value of $|\vec{a} + \vec{b} - \vec{c}|$

(1) 6 (2) $\sqrt{6}$ (3) 7 (4) $2\sqrt{6}$

Ans. (1)

Sol. $\vec{b}.\vec{a} = \vec{c}.\vec{a}$

$|\vec{a} + \vec{b} - \vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}.\vec{b} - \vec{b}.\vec{c} - \vec{a}.\vec{c})$

$= 4 + 16 + 16 + 2(\vec{a}.\vec{b} - 0 - \vec{a}.\vec{b}) = 36$

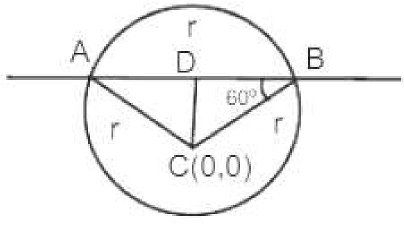
$\Rightarrow |\vec{a} + \vec{b} - \vec{c}| = 6$

3. If the line $x + 2y = 3$ cuts a chord of length r unit with the circle $x^2 = r^2$ then find r^2.

(1) $\frac{12}{5}$ (2) $\sqrt{12}$ (3) $\frac{5}{12}$ (4) $\sqrt{\frac{12}{5}}$

Ans. (1)

Sol. $AB = r, \ AD = \frac{r}{2}$



$CD = r\sin 60^\circ = \frac{\sqrt{3}r}{2}$

$$\Rightarrow \frac{|0+0-3|}{\sqrt{1^2+2^2}} = \frac{\sqrt{3}r}{1} \Rightarrow r = 2\sqrt{\frac{3}{5}} \Rightarrow r^2 = \frac{12}{5}$$

4. Find the coefficient of x^4 in the expansion of $(1 + x + x^2 + x^3)^6$

(1) 100 (2) 110 (3) 120 (4) 125

Ans. (3)

Sol. $(1 + x + x^2 + x^3)^6 = (1 + x)^6.(1 + x^2)^6$

$\Rightarrow (1 + 6x + 15x^2 + 20x^3 + 15x^4 + ...)(1 + 6x + 15x^2 + ...)$

coefficient of $x_4 = 15 + 90 + 15 = 120$

5. If the mean and standard deviation of 5, 3, 7, a, b are 5 and 2 respectively, then a and b are roots of equation.

(1) $x^2 - 10x + 18 = 0$ (2) $x^2 - 20x + 18 = 0$ (3) $x^2 - 20x + 19 = 0$ (4) $x^2 - 10x + 19 = 0$

Ans. (4)

Sol. $5 + 3 + 7 + a + b = 25 \Rightarrow a + b = 10$

$$\text{S.D.} = \sqrt{\frac{5^2 + 3^2 + 7^2 + a^2 + b^2}{2} - 5^2} = 2$$

$$= \frac{a^2 + b^2 + 83}{5} - 25 = 4 \Rightarrow a^2 + b^2 = 62$$

$\Rightarrow (a + b)^2 - 2ab = 62 \quad \Rightarrow ab = 19$

So equation whose roots are a and b is $x^2 - 10x + 19 = 0$

6. There are three section A, B, C in a paper each section having 5 questions. In how many ways a student can solve exactly 5 questions taken at least one question from each section.

(1) 2200 (2) 2225 (3) 2250 (4) 2275

Ans. (3)

Sol. $A \to 5Q$ $\quad B \to 5Q$ $\quad C \to 5Q$

A_1, A_2, A_3, A_4, A_5 $\quad B_1, B_2, B_3, B_4, B_5$ $\quad C_1, C_2, C_3, C_4, C_5$

$A_1 A_2 A_3 B_1 C_1 \Rightarrow {}^3C_1 \times {}^5C_3 \times {}^5C_1 \times {}^5C_1 = 750$

$A_1 A_2 B_1 B_2 C_1 \Rightarrow {}^3C_2 \times {}^5C_2 \times {}^5C_2 \times {}^5C_1 = 1500$

$\therefore$ total = 2250

7. $\left(\dfrac{-1+\sqrt{3}i}{1-i}\right)^{30}$ simplifies to

(1) $-2^{15}i$ (2) $2^{15}i$ (3) 2^{15} (4) -2^{15}

Ans. (1)

Sol. $$\left(\frac{-1+\sqrt{3}i}{1-i}\right)^{30} = \left(\frac{2\cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)}{\sqrt{2}\left(\cos\frac{\pi}{4} - i\sin\frac{\pi}{4}\right)}\right)^{30}$$

$$\frac{2^{30}(\cos 2\pi + i\sin 20\pi)}{2^{15}\left(\cos\frac{15\pi}{2} - i\sin\frac{15\pi}{2}\right)}$$

$$\frac{2^{15}(1+0i)}{(0+i)} = -2^{15}i$$

8. If the lines $x - y = a$ and $x + y = b$ are tangents for $y = x^2 - 3x + 2$ then $\dfrac{a}{b} =$

Ans. (2)

Sol. $y = x^2 - 3x + 2$, $\quad x + y = a_1 x - y = b$

$2x_1 - 0 = 31$ $\quad 2x_2 - 3 = -1$

$x_1 = 2$ $\quad x_2 = 1$

$x_1 = 4 - 6 + 2 = 0$ $\quad x_2 = 0$

$(2, 0)$ $\quad (1, 0)$

$a = 2$ $\quad b = 1$

$\therefore \frac{a}{b} = \frac{2}{1} = 2$

9. Let $y_1 = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ and $y_2 = \tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right)$ then $\frac{dy_1}{dy_2} =$

(1) $\frac{\sqrt{1-x^2}}{2(1+x^2)}$ (2) $\frac{\sqrt{1-x_2}}{4(1+x^2)}$ (3) $\frac{1}{(1+x^2)\sqrt{1-x^2}}$ (4) $\frac{1}{4(1+x^2)\sqrt{1-x^2}}$

Ans. (2)

Sol. Let $x = \tan\theta$

$$y_1 = \tan^{-1}\left(\frac{\sec\theta - 1}{\tan\theta}\right) = \tan^{-1}\left(\tan\frac{\theta}{2}\right) = \frac{\theta}{2} = \frac{1}{2}\tan^{-1}x$$

$$x = \sin\phi,\ y_2 = \tan^{-1}\left(\frac{2\sin\phi\cos\phi}{\cos 2\phi}\right) = \tan^{-1}(\tan 2\phi) = 2\phi = 2\sin^{-1}x$$

$$\frac{dy_1}{dy_2} = \frac{dy_1/dx}{dy_2/dx} = \frac{\frac{1}{(1+x^2)}.\frac{1}{2}}{2.\frac{1}{\sqrt{1-x^2}}}$$

$$= \frac{\sqrt{1-x^2}}{4(1+x^2)}$$

10. In a G.P. sum of 2nd, 3rd and 4th term is 3 and that of 6th, 7th and 8th term is 243 then S_{50}=

(1) $\frac{3^{50}+1}{26}$ (2) $\frac{3^{50}-1}{13}$ (3) $\frac{3^{50}-1}{26}$ (4) $\frac{3^{49}-1}{26}$

Ans. (3)

Sol. Let a, ar, ar^2, ... G.P.

$T_2 + T_2 + T_4 = 3$ $\quad \Rightarrow ar(1 + r + r^2) = 3$ .... (i)

$T_6 + T_7 + T_8 = 243$ $\quad \Rightarrow ar^5(1 + r + r^2) = 243$ .... (ii)

by (i) and (ii)

$r^4 = 81$ $\quad \Rightarrow r = 3$

$\therefore a = \frac{1}{13}$

$$S_{50} = \frac{a(r^{50}-1)}{r-1} = \frac{3^{50}-1}{26}$$

11. $\int \frac{\cos\theta}{7 + \sin\theta - 2\cos^2\theta} d\theta$ is equal to

(1) $\frac{2}{\sqrt{39}}\tan^{-1}\left(\frac{2\sin\theta+1}{\sqrt{39}}\right)+C$ (2) $\frac{2}{\sqrt{39}}\tan^{-1}\left(\frac{4\sin\theta+1}{\sqrt{39}}\right)+C$

(3) $\frac{4}{\sqrt{39}}\tan^{-1}\left(\frac{4\sin\theta+1}{\sqrt{39}}\right)+C$ (4) $\frac{4}{\sqrt{39}}\tan^{-1}\left(\frac{2\sin\theta+1}{\sqrt{39}}\right)+C$

Ans. (2)

Sol. $I = \int \frac{\cos\theta}{7+\sin\theta-2(1-\sin^2\theta)}d\theta$

$= \int \frac{\cos\theta}{2\sin^2\theta+\sin\theta+5}d\theta$

$\sin\theta = t \qquad \Rightarrow \qquad \cos\theta.d\theta = dt$

$I = \int \frac{dt}{2t^2+t+5} = \frac{1}{2}\int \frac{dt}{\left(t+\frac{1}{4}\right)^2+\frac{39}{16}}$

$\frac{1}{2}.\frac{1}{\frac{\sqrt{39}}{4}}\tan^{-1}\left(\frac{t+\frac{1}{4}}{\frac{\sqrt{39}}{4}}\right)+C$

$\frac{2}{\sqrt{39}}\tan^{-1}\left(\frac{4\sin\theta+1}{\sqrt{39}}\right)+C$

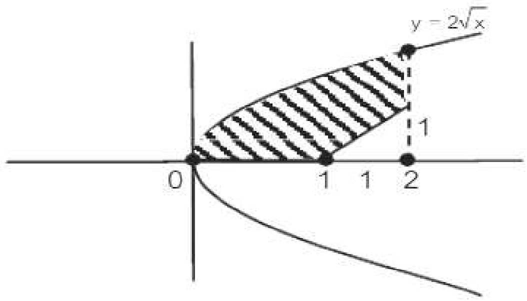
12. The area enclosed by $[x].(x-1) \le y \le 2\sqrt{x}$ from $x = 0$ to 2 where $[x]$ is the greatest integer less than or equal to x., is equal to

(1) $\frac{8\sqrt{2}}{3}+\frac{1}{2}$ (2) $\frac{8\sqrt{2}}{3}$ (3) $\frac{8\sqrt{2}}{3}-\frac{1}{2}$ (4) $\frac{8}{3}-\frac{1}{\sqrt{2}}$

Ans. (3)

Sol. $y = [x]\,(x - 1)$

$= \begin{cases} 0 & 0 \le x \le 1 \\ x-1 & 1 \le x \le 2 \end{cases}$



$\text{Area} = \int_0^2 2\sqrt{x}.dx - \frac{1}{2}(1)(1) = \left(\frac{4x^{3/2}}{3}\right)_0^2 - \frac{1}{2} = \frac{8\sqrt{2}}{3}-\frac{1}{2}$

13. If x +a = y + b + 1 = z + c then the value of $\begin{vmatrix} x & a+y & a+x \\ y & b+y & b+y \\ z & c+y & c+z \end{vmatrix}$ is

(1) y (a – b) (2) y (b – c) (3) y (c – a) (4) 0

Ans. (3)

Sol. Given x + a = y + b + 1 = z + c

$$\text{Now}\begin{vmatrix} x & a+y & a+x \\ y & b+y & b+y \\ z & c+y & c+z \end{vmatrix} = \begin{vmatrix} x & a+y & a \\ y & b+y & b \\ z & c+y & c \end{vmatrix} (C_3 \to C_3 - C_1)$$

$$= \begin{vmatrix} x & y & a \\ y & y & b \\ z & y & c \end{vmatrix} (C_2 \to C_2 - C_3)$$

$$= y\begin{vmatrix} x & 1 & a \\ y & 1 & b \\ z & 1 & c \end{vmatrix}$$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$y.\begin{vmatrix} x & 1 & a \\ y-x & 0 & b-a \\ z-x & 0 & c-a \end{vmatrix}$$

$$= -y(c-a)(y-x+b-a) = y(c-a)$$

14. If $\log_{7^{\frac{1}{2}}} x + \log_{7^{\frac{1}{3}}} x + \log_{7^{\frac{1}{4}}} + x......20$ time $= 460$ than $x = ?$

Ans. (49)

Sol. Given $\log_{7^{\frac{1}{2}}} x + \log_{7^{\frac{1}{3}}} x + \log_{7^{\frac{1}{4}}} x + ...20 \text{ times} = 460$

$\Rightarrow (2 + 3 + 4 + ...+21)\log_7 x = 460$

$\Rightarrow \frac{20}{2}(2+21)\log_7 x = 460$

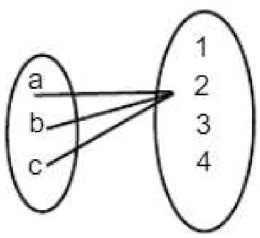
$\Rightarrow \log_7 x = 2$

$\Rightarrow x = 49$

15. A function $f : A \to B$ where $A = \{a, b, c\}$, $B = \{1, 2, 3, 4\}$. How many function can be defined from A to B which are not one-one such that $2 \in f(A)$

Ans. (19)

Sol. only '2' in range $\to$ 1 function



one element out of 1, 3, 4 is in range with '2'

number fo ways $= {}^3C_1.\frac{3!}{2!.1!}.2! = 18$

(select one from 1, 3, 4 and distribute among a, b, c)

Total function = 1 + 18 = 19

16. If the system of equation $x + y + z = 0$, $x + 3y + k^2 z = 0$ and $x + 2y + z = 0$ have a non zero solution then the value of $y + \frac{x}{z}$ is

(1) 1 (2) 0 (3) –1 (4) 2

Ans. (3)

Sol. $\left.\begin{array}{l} x + y + z = 0 \\ x + 3y + k^2 z = 0 \\ x + 2y + z = 0 \end{array}\right\}$ has a non zero solution

$$\therefore \begin{vmatrix} 1 & 1 & 1 \\ 1 & 3 & k^2 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

$\Rightarrow y = 0, x + z = 0 \quad \Rightarrow \frac{x}{z} = -1$

$\therefore y + \frac{x}{z} = -1$

17. If $y = mx + C$ is a common tangent of circle $x^2 + y^2 = 3$ and hyperbola $\frac{x^2}{64} - \frac{y^2}{100} = 1$ then which of the following statement is true:

(1) $8m = 4$ (2) $61C^2 = 492$ (3) $4C^2 = 369$ (4) $8m + 5 = 0$

Ans. (2)

Sol. Tangent of circle $\Rightarrow C^2 = 3(1 + m^2)$

and tangent of hyperbola $\Rightarrow C^2 = 64m^2 - 100$

$\Rightarrow 3(1 + m^2) = 64m^2 - 100$

$\Rightarrow 61m^2 = 103$

$\Rightarrow m^2 = \frac{103}{61}$

$\Rightarrow C^2 = 3\left(1 + \frac{103}{61}\right) = \frac{492}{61}$

gravity
Orienting Intelligence

JEE MAIN 2020
PHASE 1

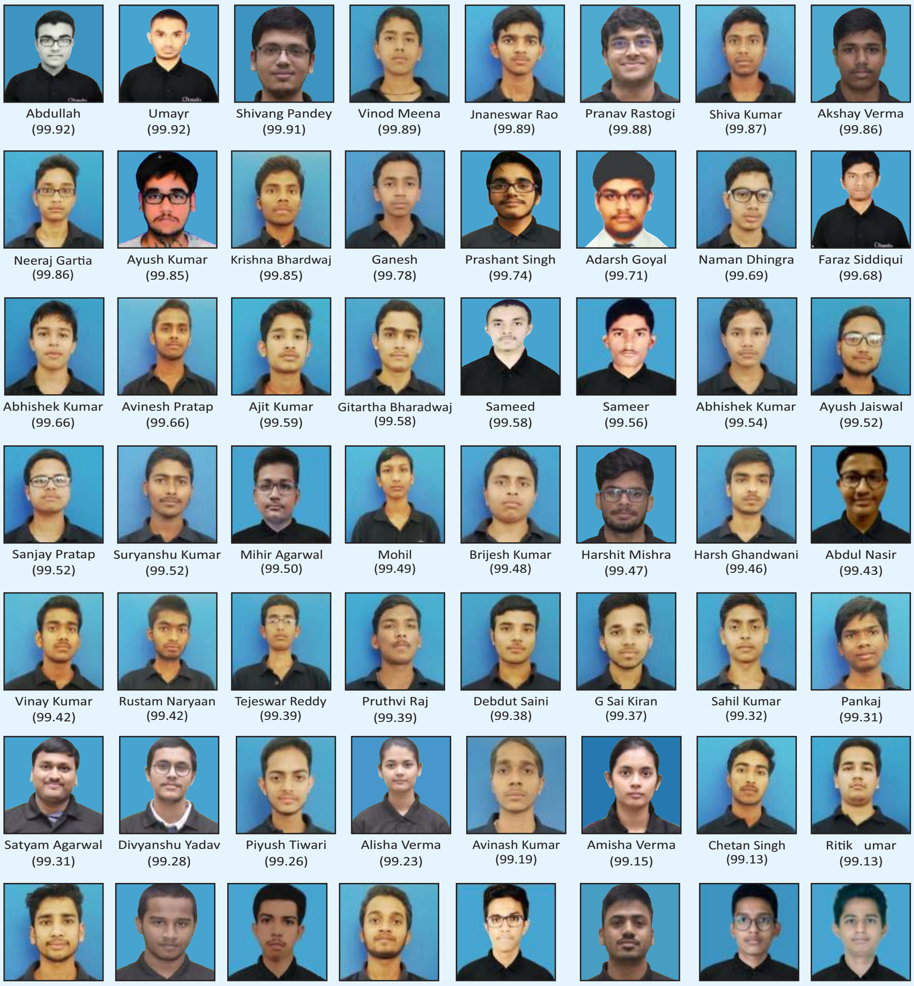
JEE Main - 2020 | Best Result in U.P.

Aditya Pandey
Percentile
99.936
City Topper
Application No. 200310320565
DOB - 23-12-2002

SCHOOL INTEGRATED PROGRAM (SIP)

Tradition of Gravity Continues, Once Again Historical Result, 100% Students Cracked JEE Main (Based on Last Yr Cut off)

65 Students Above 99 Percentile
145 Students Above 98 Percentile
208 Students Above 97 Percentile



Abdullah (99.92)	Umayr (99.92)	Shivang Pandey (99.91)	Vinod Meena (99.89)	Jnaneswar Rao (99.89)	Pranav Rastogi (99.88)	Shiva Kumar (99.87)	Akshay Verma (99.86)
Neeraj Gartia (99.86)	Ayush Kumar (99.85)	Krishna Bhardwaj (99.85)	Ganesh (99.78)	Prashant Singh (99.74)	Adarsh Goyal (99.71)	Naman Dhingra (99.69)	Faraz Siddiqui (99.68)
Abhishek Kumar (99.66)	Avinesh Pratap (99.66)	Ajit Kumar (99.59)	Gitartha Bharadwaj (99.58)	Sameed (99.58)	Sameer (99.56)	Abhishek Kumar (99.54)	Ayush Jaiswal (99.52)
Sanjay Pratap (99.52)	Suryanshu Kumar (99.52)	Mihir Agarwal (99.50)	Mohil (99.49)	Brijesh Kumar (99.48)	Harshit Mishra (99.47)	Harsh Ghandwani (99.46)	Abdul Nasir (99.43)
Vinay Kumar (99.42)	Rustam Naryaan (99.42)	Tejeswar Reddy (99.39)	Pruthvi Raj (99.39)	Debdut Saini (99.38)	G Sai Kiran (99.37)	Sahil Kumar (99.32)	Pankaj (99.31)
Satyam Agarwal (99.31)	Divyanshu Yadav (99.28)	Piyush Tiwari (99.26)	Alisha Verma (99.23)	Avinash Kumar (99.19)	Amisha Verma (99.15)	Chetan Singh (99.13)	Ritik umar (99.13)
Shubham Kumar (99.10)	Randheer (99.08)	Aryansh Tripathi (99.06)	Saurabh Maurya (99.03)	Mohammad Affan (99.03)	Pradumna Awasthi (99.02)	Harsh (99.02)	Abhay (99.01)

2020

80 Out of 80
Cracked JEE Main

We had three Batches of 55, 15 and 10.

Many Top Ranks are from these Batches

2019

79 Out of 80 in JEE Main	50 Out of 79 in JEE Adv.

2018

83 Out of 85 in JEE Main	62 Out of 83 in JEE Adv.

2017

80 Out of 85 in JEE Main	63 Out of 80 in JEE Adv.

2016

39 Out of 40 in JEE Main	31 Out of 39 in JEE Adv.

Selections Engineering 2019

gravity
Orienting Intelligence


Tarun
194 AIR (General)


Aniket Agarwal
337 AIR (General)


Shubh Sahu
494 AIR (General)


Shlok Nemani
497 AIR (General)

50 out of 79 Cracked JEE Advanced from SIP (School Integrated Program)

4 Ranks under 500 (General Category) | 2 Ranks under 10 (Reserved Category)

126 Selections in JEE Advanced | 61 Students above 99 Percentile in JEE Main 2019


Sanjana
AIR - 3*


Akash
AIR - 4*


Priyanka
AIR - 68*

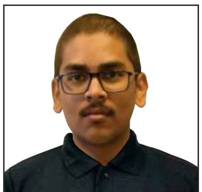

Bibek Lakra
AIR - 150*


Neha Raj
AIR - 177*


Arindam
AIR - 809 (General EWS)


Priyam
AIR - 1378 (General)


Mihir Chawla
AIR - 2237 (General)


Madhur Kumar
AIR - 2382 (General)


Manish Kumar
AIR - 2388 (General)


Saumya Raj
AIR - 2656 (General)


Raghav
AIR - 2659 (General)


Ritveek
AIR - 2709 (General)


Vanshaj
AIR - 2787 (General)


Subir Gupta
AIR - 2881 (General)


Aryan Rastogi
AIR - 3167 (General)


Devansh
AIR - 3600 (General)


Abhisht Bose
AIR - 3784 (General)