JEE Main (Phase-II) 2020

Memory Based Questions & Solutions

SUBJECT

MATHEMATICS

Date: 06 September, 2020 (Shift-2)

Time: 3 PM to 6 PM

1. If α, β are the roots of equation $2x(2x+1)=1$ then β = ?

(1) α^2 (2) $-2\alpha(\alpha+1)$ (3) $2\alpha(\alpha+1)$ (4) $\alpha(\alpha-1)$

Ans. (2)

Sol. Given equation is $2x(2x+1)=1 \Rightarrow 4x^2+2x-1=0$ ..... (1)

roots of equation (1) are α and β

$$\therefore\ \alpha+\beta=-\frac{1}{2} \qquad \Rightarrow \beta=-\frac{1}{2}-\alpha \qquad \text{..... (2)}$$

and

$$4\alpha^2+2\alpha-1=0 \qquad \Rightarrow \alpha^2=\frac{1}{4}-\frac{\alpha}{2} \qquad \text{.....(3)}$$

now

$$-2\alpha(\alpha+1)=-2\alpha^2-2\alpha$$

$$=-2\left(\frac{1}{4}-\frac{\alpha}{2}\right)-2\alpha=-\frac{1}{2}-\alpha=\beta$$

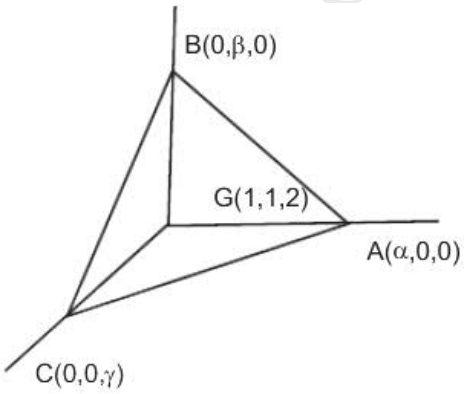
2. A plane intersects the x, y, z axis at A, B, C respectively. If G(1, 1, 2) is centroid of ΔABC, then the equation of the line perpendicular to plane and passing through G is

(1) $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-2}{2}$ (2) $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-2}{2}$

(3) $\frac{x-1}{2}=\frac{y-1}{2}=\frac{z-2}{1}$ (4) $\frac{x-1}{3}=\frac{y-2}{3}=\frac{z-2}{6}$

Ans. (3)

Sol. Let $A(\alpha, 0, 0), B(0, \beta, 0), C(0, 0, \gamma)$ then $G\left(\frac{\alpha}{3},\frac{\beta}{3},\frac{\gamma}{3}\right)\equiv(1, 1, 2)$



$\alpha=3, \beta=3, \gamma=6$

$\therefore$ equation of plane is $\frac{x}{\alpha}+\frac{y}{\beta}+\frac{z}{\gamma}=1$

$\Rightarrow \frac{x}{3}+\frac{y}{3}+\frac{z}{6}=1$

$\Rightarrow 2x+2y+z=6$

$\therefore$ required line $\frac{x-1}{2}=\frac{y-1}{2}=\frac{z-2}{1}$

3. Total number of words (with or without meaning) from letters of word 'LETTER' if no two vowels are together

(1) 100 (2) 110 (3) 120 (4) 180

Ans. (3)

Sol. Constants are L, T, T, R

Vowels are E, E,

Total number of words (with or without meaning) from letters of word 'LETTER' = $\frac{6!}{2!2!}=180$

Total number of words (with or without meaning) from letters of word 'LETTER' is vowels are together

$\frac{5!}{2!}=60$

$\therefore$ Required = 180 - 60 = 120

4. If the constant terms in the expansion of $\left(\sqrt{x}-\frac{K}{x^2}\right)^{10}$ is 405 then |k| = ?

(1) 9 (2) $\frac{9}{2}$ (3) 3 (4) 6

Ans. (3)

Sol. $T_{r+1}={}^{10}C_{r.}\left(\frac{-K}{x^2}\right)^{r}\left(\sqrt{x}\right)^{10-r}$

$={}^{10}C_{r.}(-K)^{r}.\ x^{5-\frac{5r}{2}}$

for constant term $\Rightarrow\ 5-\frac{5r}{2}=0\ \Rightarrow r=2$

$\Rightarrow\ T_3={}^{10}C_2.K^2.=405$

$\Rightarrow\ \frac{10(9)}{2}K^2=405$

$\Rightarrow K^2=9\ \Rightarrow\ |K|=3$

5. Centre of a circle passing through point (0, 1) and touching the curve $y = x^2$ at (2, 4) is

(1) $\left(\frac{16}{5},\frac{53}{10}\right)$ (2) $\left(\frac{-16}{5},\frac{53}{10}\right)$ (3) $\left(\frac{-16}{5},\frac{-53}{10}\right)$ (4) $\left(\frac{16}{5},\frac{-53}{10}\right)$

Ans. (2)

Sol. $y = x^2$, (2, 4)

tangent at (2, 4) is

$\frac{1}{2}(y+4)=2x$

$y + 4 = 4x\ \Rightarrow\ 4x - y - 4 = 0$

Equation of circle $(x-2)^2+(y-4)^2+\lambda(4x-y-4)=0$

it passes through (0, 1)

$\therefore\ 4+9+\lambda(0-1-4)=0$

$13=5\lambda\Rightarrow\lambda=\frac{13}{5}$

$\therefore$ circle is $x^2-4x+4+y^2-8y+16+\frac{13}{5}(4x-y-4)=0$

$\Rightarrow x^2+y^2+\left(\frac{52}{5}-4\right)x-\left(8+\frac{13}{5}\right)y+20-\frac{52}{5}=0$

$\Rightarrow x^2+y^2+\frac{32}{5}x-\frac{53}{5}y+\frac{48}{5}=0$

$\because$ centre is $\left(-\frac{16}{5},\frac{53}{10}\right)$

6. If $A=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$ and $\beta=A^4+A$ then determinant of β = ?

(1) $2-2.\cos3\theta$ (2) $2+3.\cos2\theta$ (3) $3+\cos2\theta$ (4) $2+2.\cos3\theta$

Ans. (4)

Sol. $A^2=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$

$A^2=\begin{bmatrix}\cos2\theta & \sin2\theta\\ -\sin2\theta & \cos2\theta\end{bmatrix}$

$\Rightarrow A^4=\begin{bmatrix}\cos4\theta & \sin4\theta\\ -\sin4\theta & \cos4\theta\end{bmatrix}$

$\beta=\begin{bmatrix}\cos4\theta & \sin4\theta\\ -\sin4\theta & \cos4\theta\end{bmatrix}+\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$

$=\begin{bmatrix}\cos4\theta+\cos\theta & \sin4\theta+\sin\theta\\ -(\sin4\theta+\sin\theta) & \cos4\theta+\cos\theta\end{bmatrix}$

$|\beta|=(\cos4\theta+\cos\theta)^2+(\sin4\theta+\sin\theta)^2$

$=2+2(\cos4\theta.\cos\theta+\sin4\theta+\sin\theta)$

$=2+2\cos(4\theta-\theta)$

$=2+2.\cos3\theta$

7. Contrapositive of "If n^3-1 is even then n is odd" is

(1) If n is odd then n^3-1 is not even

(2) If n is not odd then n^3-1 is even

(3) If n is not odd then n^3-1 is not even

(4) If n is odd then n^3-1 is even

Ans. (3)

Sol. P : n^3-1 is even, q : n is odd

contrapositive of $p\rightarrow q=\sim q\rightarrow\sim p$

$\Rightarrow$ "If n is not odd then n^3-1 is not even"

8. If a_1, a_2, a_3 ..... a_n and b_1, b_2, b_3 ..... b_n are two arithmetic progression with common difference of 2nd is two more than that of first and $b_{100}=a_{70}$, $a_{100}=-399$, $a_{410}=-159$ then the value of b_1 is

(1) – 51 (2) – 61 (3) – 81 (4) 81

Ans. (3)

Sol. Let $a_1 a_1+d$, a_1+2d ..... first A.P.

$a_{40}=a_1+39d=-159$ ..... (1)

$a_{100} = a_1 + 99d = -339$ ..... (2)

from equation (1) and (2)

$d = -4$, $a_1 = -3$

now

$b_{100} = a_{70}$

$\Rightarrow b_1 + 99D = a_1 + 69d$

$b_1 + 99 \times -2 = -3 + 69 \times -4$ (According to question $D = d + 2$)

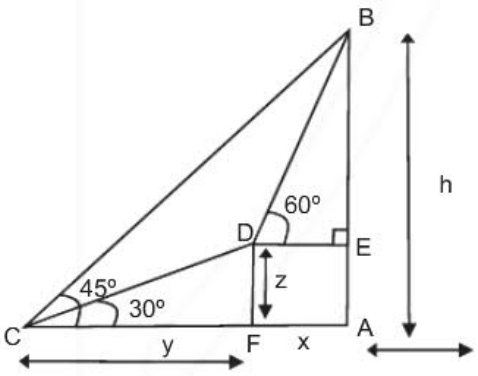
$\Rightarrow b_1 = -81$

9. If the angle of elevation of the tap of a summit is 45^o and a person climbs at an inclination of 30^o upto 1 km. Where the angle of elevation of top becomes 60^o, then height of the summit is

(1) $\frac{1}{\sqrt{3}+1}$km (2) $\frac{\sqrt{3}-2}{2}$km (3) $\frac{\sqrt{3}+1}{2}$km (4) $\frac{3}{\sqrt{3}-1}$km

Ans. (3)

Sol.



In ΔCDF

$\sin 30^o = \frac{z}{1}[CD = 1km(given)]$

$z = \frac{1}{2}$ ..... (1)

$\cos 30^o = \frac{y}{1} \Rightarrow = \frac{\sqrt{3}}{2}$

now in ΔABC

$\tan 45^o = \frac{h}{x+y}$

$\Rightarrow h = x + y$

$\Rightarrow x = h - \frac{\sqrt{3}}{2}$ ..... (2)

now

In ΔBDE ,

$\tan 60^o = \frac{h-z}{x}$

$\sqrt{3}x = h - \frac{1}{2}$

$\Rightarrow \sqrt{3}\left(h - \frac{\sqrt{3}}{2}\right) = h - \frac{1}{2}$

$\Rightarrow \sqrt{3}-1h=1$

$h=\frac{1}{\sqrt{3}-1}=\frac{\sqrt{3}+1}{2}km$

10. If ${}^nC_0$, ${}^nC_1$, ${}^nC_2$,.....${}^nC_n$ are frequencies of n + 1 observations 1, 2, 2^2,.....2^n such that mean is $\frac{729}{2^n}$ then value of n is

Ans. 06.00

Sol.

x_i(observation)	1	2	2^2	2^n
f_i(frequency)	${}^nC_0$	${}^nC_1$	${}^nC_2$	${}^nC_n$

$\overline{x}=\frac{\sum f_i x_i}{\sum f_i}$

$\frac{1\,x\,{}^nC_0+2\,x\,{}^nC_1+2^2\,x\,{}^nC_2.....2^n\,x\,{}^nC_n}{{}^nC_0+{}^nC_1+{}^nC_2.....+{}^nC_n}=\frac{3^n}{2^n}=\frac{729}{2^n}$

$\Rightarrow 3^n=3^6$

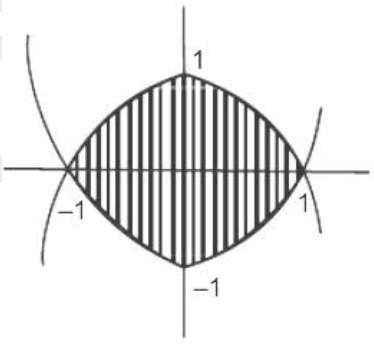
$\Rightarrow n=6$

11. Area bounded by curves $y=x^2-1$ and $y=1-x^2$ is

(1) $\frac{1}{3}$ (2) $\frac{8}{3}$ (3) $\frac{4}{3}$ (4) $\frac{2}{3}$

Ans. (2)

Sol.



Given curves are $y=x^2-1$ and $y=1-x^2$ so intersection point are $(\pm1, 0)$

bounded area = $4\int_0^1(1-x^2)dx=4\left[x-\frac{x^3}{3}\right]_0^1=4\left(1-\frac{1}{3}\right)=\frac{8}{3}$sq. units

Gravity
Orienting Intelligence

JEE MAIN 2020
PHASE 1

JEE Main - 2020

Best Result in U.P.

Aditya Pandey

Percentile

99.936

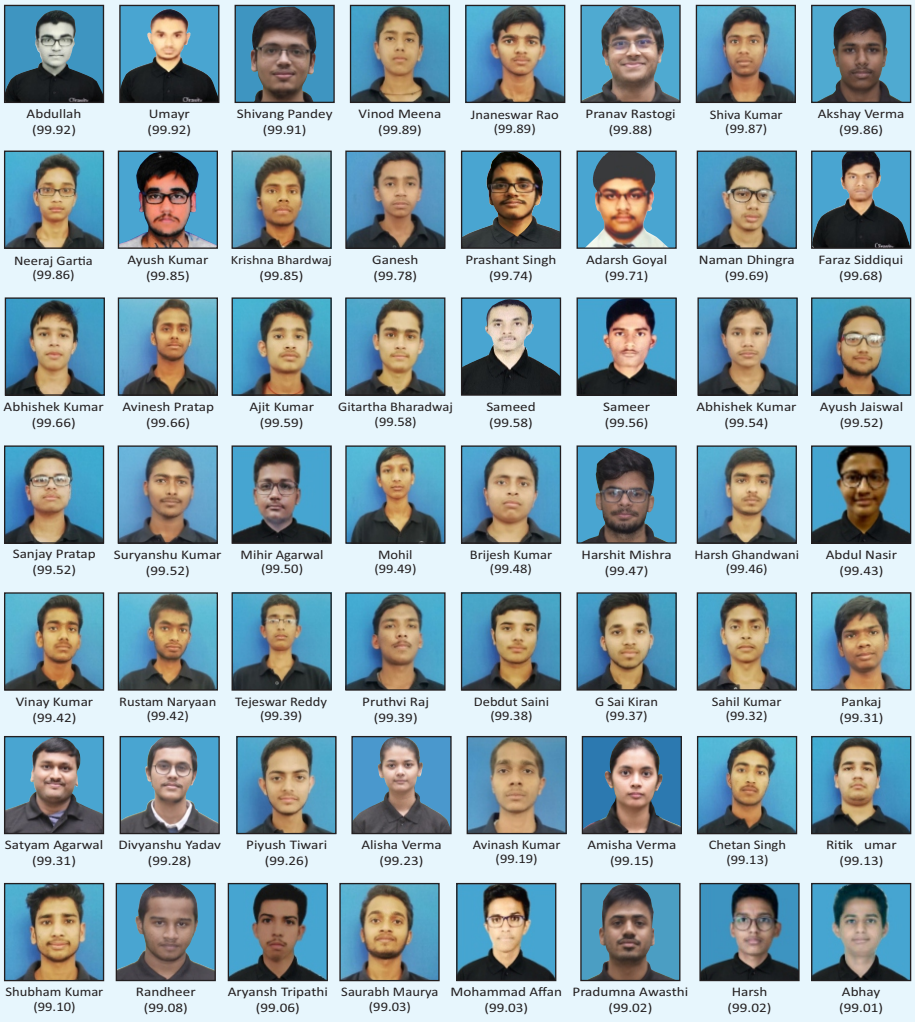
City Topper

Application No. 200310320565
DOB - 23-12-2002

65 Students Above 99 Percentile

145 Students Above 98 Percentile

208 Students Above 97 Percentile



SCHOOL INTEGRATED PROGRAM (SIP)

Tradition of Gravity Continues,
Once Again Historical Result,
100% Students Cracked
JEE Main
(Based on Last Yr Cut off)

2020

80 Out of 80
Cracked JEE Main

We had three Batches
of 55, 15 and 10.

Many Top Ranks are
from these Batches

2019

79 Out of 80 in JEE Main	50 Out of 79 in JEE Adv.

2018

83 Out of 85 in JEE Main	62 Out of 83 in JEE Adv.

2017

80 Out of 85 in JEE Main	63 Out of 80 in JEE Adv.

2016

39 Out of 40 in JEE Main	31 Out of 39 in JEE Adv.

Selections Engineering 2019

gravity
Orienting Intelligence

194 AIR (General)

Tarun

337 AIR (General)

Aniket Agarwal

494 AIR (General)

Shubh Sahu

497 AIR (General)

Shlok Nemani

50 out of 79 Cracked JEE Advanced from SIP (School Integrated Program)

4 Ranks under 500 (General Category) | 2 Ranks under 10 (Reserved Category)

126 Selections in JEE Advanced | 61 Students above 99 Percentile in JEE Main 2019

Sanjana

AIR - 3*

Akash

AIR - 4*

Priyanka

AIR - 68*

Bibek Lakra

AIR - 150*

Neha Raj

AIR - 177*

Arindam

AIR - 809
(General EWS)

Priyam

AIR - 1378
(General)

Mihir Chawla

AIR - 2237
(General)

Madhur Kumar

AIR - 2382
(General)

Manish Kumar

AIR - 2388
(General)

Saumya Raj

AIR - 2656
(General)

Raghav

AIR - 2659
(General)

Ritveek

AIR - 2709
(General)

Vanshaj

AIR - 2787
(General)

Subir Gupta

AIR - 2881
(General)

Aryan Rastogi

AIR - 3167
(General)

Devansh

AIR - 3600
(General)

Abhisht Bose

AIR - 3784
(General)