JEE Main (Phase-II) 2020

Memory Based Questions & Solutions

SUBJECT

MATHEMATICS

Date: 02 September, 2020 (Shift-2)

Time: 3 PM to 6 PM

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1. Find the imaginary part of $\left(\left(3+2\sqrt{-54}\right)^{1/2}-\left(3-2\sqrt{-54}\right)^{1/2}\right)$

(a) $-\sqrt{6}$ (b) $-2\sqrt{6}$ (c) $\sqrt{6}$ (d) 6

Ans. (b)

Sol : $\left|3+2\sqrt{-54}\right|=\sqrt{9+216}=5$

$\Rightarrow \left(3+2\sqrt{-54}\right)^{1/2}=\pm\left(\sqrt{\frac{15+3}{2}}+i\sqrt{\frac{15-3}{2}}\right)$

$=\pm\left(3+i\sqrt{6}\right)$

and $\left(3-2\sqrt{-54}\right)^{1/2}=\pm\left(3-i\sqrt{6}\right)$

Hence $\left\{\left(3+2\sqrt{-54}\right)^{1/2}-\left(3-2\sqrt{-54}\right)^{1/2}\right\}$

$=\pm 2i\sqrt{6}$ or ± 6

Hence imaginary part = $-2\sqrt{6}$

2. If a, b, c $\in$ R such that $a^3+b^3+c^3=2$ and $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}=0$ then find abc

(a) $\frac{2}{3}$ (b) $-\frac{2}{3}$ (c) $\frac{1}{3}$ (d) 1

Ans. (a)

Sol : $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}=0 \Rightarrow 3abc - a^3 - b^3 - c^3 = 0$

$\Rightarrow abc = \frac{a^3+b^3+c^3}{3}=\frac{2}{3}$

3. The ratio of three consecutive binomial coefficient in the expansion of $(1+x)^n$ is 2 : 5 : 12 find n

(a) 120 (b) 34 (c) 118 (d) 35

Ans. (c)

Sol : ${}^nC_{r-1} : {}^nC_{r+1} = 2 : 5 : 12$

$\Rightarrow \frac{{}^nC_r}{{}^nC_{r-1}}=\frac{5}{2}$ and $\frac{{}^nC_{r+1}}{{}^nC_r}=\frac{12}{5}$

$\Rightarrow \frac{n-r+1}{r}=\frac{5}{2}$ and $\frac{n-r}{r+1}=\frac{12}{5}$

$\Rightarrow 2n - 7r + 2 = 0$ and $5n - 17r - 12 = 0$

on solving n = 118 and r = 34

4. There are n stations in a circular path. Two consecutive stations are connected by blue line and two non-consecutive stations are connected by red line. If no. of red lines is equal to 99 times number of blue line then value of n is

(a) 201 (b) 200 (c) 199 (d) 202

Ans. (a)

Sol : Two consecutive statins = n

Two non-consecutive stations = ${}^nC_2$ - n

${}^nC_2 - n = 90 \Rightarrow \frac{n(n-1)}{2} - n = 99n$

$\Rightarrow \frac{n^2-1}{2} = 100n$

$\Rightarrow n^2 = 201\,n \Rightarrow n = 201$

5. $\int_1^2 \left|2x - [3x]\right| dx = ?$ (where [.] denotes greatest integer function)

(a) 1 (b) 3 (c) 2 (d) 4

Ans. (a)

Sol : $\int_1^2 \left|2x-[3x]\right| dx \Rightarrow \int_1^2 \left|\{3x\}-x\right| dx = \int_1^2 \left|x-\{3x\}\right| dx \Rightarrow \int_1^2 x dx - \int_1^2 \{3x\} dx$

$\Rightarrow \left(\frac{x^2}{2}\right)_1^2 - 3.\int_1^{1/3} 3x dx = \left(\frac{4}{2}-\frac{1}{2}\right) - 9\left(\frac{x^2}{2}\right)_0^{1/3}$

$\Rightarrow \frac{3}{2} - \frac{9}{2}\left((1/3)^2 - 0^2\right) = 1$

6. If $x^2 - y^2 \sec^2\theta = 10$ be a hyperbola and $x^2\sec^2\theta + y^2 = 5$ be an ellipse such that the eccentricity of hyperbola $= \sqrt{5}$ eccentricity of ellipse then find the length of latus rectum of ellipse

(a) $\frac{4\sqrt{5}}{3}$ (b) $\frac{4}{3\sqrt{5}}$ (c) $\frac{20\sqrt{5}}{3}$ (d) $\sqrt{30}$

Ans. (a)

Sol : $\frac{x^2}{10} - \frac{y^2}{10cos^2\theta} = 1 \Rightarrow e_H = \sqrt{1+\cos^2\theta}$ and $\frac{x^2}{5\cos^2\theta} + \frac{y^2}{5} = 1 \Rightarrow e_E = \sqrt{1-\cos^2\theta} = \sin\theta$

as given $e_H = \sqrt{5}e_E$

$\Rightarrow 1 + \cos^2\theta = 5\sin^2\theta$

$\Rightarrow \cos^2\theta = \frac{2}{3}$

Now length of L.R. of ellipse $= \frac{10\cos^2\theta}{\sqrt{5}} = \frac{20}{3\sqrt{5}} = \frac{4\sqrt{5}}{3}$

7. $\lim_{x\to 0} \tan\left(\frac{\pi}{4}+x\right)^{1/x} = ?$

(a) e (b) e^2 (c) e^4 (d) $\frac{1}{e}$

Ans. (b)

Sol : $e^{\lim_{x\to 0}\frac{1}{x}\left(\tan\left(\frac{\pi}{4}+x\right)-1\right)} = e^{\lim_{x\to 0}\frac{1}{x}\left(\frac{1+\tan x}{1-\tan x}-1\right)}$

$\Rightarrow e^{\lim_{x\to 0}\frac{2\tan x}{x(1-\tan x)}} = e^{\lim_{x\to 0}\left(\frac{tanx}{x}\right)\left(\frac{2}{1-\tan x}\right)}$

$\Rightarrow e^{(1)\left(\frac{2}{1-0}\right)} = e^2$

8. If f(x) be a quadratic polynomial such that f(x) = 0 has a root 3 and f(2) + f(-1) = 0 then other root lies in

(a) (-1,0) (b) (0, 1) (c) (-2, 1) (d) (1, 2)

Ans. (a)

Sol : Let $f(x) = ax^2 + bx + c$

f(b) + f(-1) = 0

$\Rightarrow$ 5a + b + 2c = 0

and f(c) = 0 $\Rightarrow$ 9a + 3b + c = 0 $\Rightarrow \frac{a}{-5} = \frac{b}{13} = \frac{c}{6}$

product of root $\alpha\beta = \frac{c}{a} = -\frac{6}{5}$

and $\alpha = 3 \Rightarrow \beta = -\frac{2}{5} \in (-1,0)$

9. If a curve y = f(x) satisfy the differential equation $2x^2dy = (2xy + y^2)dx$ and passes (1, 2) then find f(1/2)

(a) $\frac{1}{1+\ln 2}$ (b) $\frac{1}{1-\ln 2}$ (c) $\frac{2}{1-\ln 2}$ (d) $\frac{2}{1+\ln 2}$

Ans. (a)

Sol : $\frac{dy}{dx} = \frac{y}{x} + \frac{y^2}{2x^2}$

$= y^{-2}\frac{dy}{dx} - \frac{1}{y}.\frac{1}{x} = \frac{1}{2x^2}$

Put $-\frac{1}{y} = t \Rightarrow \frac{1}{y^2}\frac{dy}{dx} = \frac{dy}{dx} \Rightarrow \frac{dt}{dx} + \left(\frac{1}{x}\right)t = \frac{1}{2x^2}$

Linear differntial equation

I.F. $e^{\int \frac{1}{x}dx} = e^{\ln x} = x$

so solution of the linear differential equation is

$tx = \int \frac{1}{2x^2}.\text{x}\, dx + C$

$\Rightarrow -\frac{x}{y} = \frac{1}{2}\ln x + C$

The curve passes through (1,2)

$\Rightarrow \frac{1}{2} = \frac{1}{2}\ln 1 + C \Rightarrow C = -\frac{1}{2}$

Hence $-\frac{x}{y} = \frac{1}{2}\ln x. - \frac{1}{2}$

or $\frac{x}{y} = \frac{1-\ln x}{2} \Rightarrow y = \frac{2x}{1-\ln x} \Rightarrow f\left(\frac{1}{2}\right) = \frac{2\times\frac{1}{2}}{1-\ln\frac{1}{2}} = \frac{1}{1+\ln 2}$

10. If A, B, C are three pairwise independent event such that P(A $\cap$ B $\cap$ C) = 0 then $P\left(\left(B^C \cap C^C\right)/A\right)$ is equal

(a) P(C) + P(B) (b) P(C^C) + P(B) (c) P(B^C) + P(C) (d) P(C^C) - P(B)

Ans. (d)

Sol : $P\left(\left(B^C \cap C^C\right)/A\right) = \frac{P\left(A\cap\left(B^C\cap C^C\right)\right)}{P(A)}$

$= \frac{P(A) - \{P(A\cap B) + P(A\cap C) - P(A\cap B\cap C)\}}{P(A)}$

$= \frac{P(A) - P(A).P(B) - P(A).P(C) + 0}{P(A)}$

= 1 - P(B) - P(C)

= P(C^C) - P(B) or P(B^C) - P(C)

11. If sum of series (x + ka) + (x^2 + (k - 2)a) + (x^3 + (k -4)a) + ...9 terms is $\frac{x^{10} - x - 45a(x-1)}{x-1}$ then value of k is :

Ans : 03.00

Sol : Given (x + ka) + (x^2 + (k - 2)a) + (x^3 + (k -4)a) + ...9 terms

$= \frac{x\left(x^9 - 1\right)}{x-1} + \frac{9}{2}\left[2ka + (9-1)(-2a)\right]$

$= \frac{x^{10} - x}{x-1} + (9ka - 72a) = \frac{x^{10} - x + 9a(k-8)(x-1)}{x-1}$

= 9a (k - 8) = -45 a

k = 3

12. Point P divides line joining $A\left(\hat{i}+\hat{j}+\hat{k}\right)$ and $B\left(2\hat{i}+\hat{j}+3\hat{k}\right)$ in the ration $\lambda:1$ such that

$\overrightarrow{OB}.\overrightarrow{OP}-3\left|\overrightarrow{OA}\times\overrightarrow{OP}\right|^2=6$. Find λ.

Ans : 00.80

Sol : P.V. op is $\overrightarrow{OP}=\dfrac{\vec{a}+\lambda\vec{b}}{\lambda+1}$

$$\because\ \vec{b}\left(\frac{\vec{a}+\lambda\vec{b}}{\lambda+1}\right)-3\left|\vec{a}\times\left(\frac{\vec{a}+\lambda\vec{b}}{\lambda+1}\right)\right|=6$$

$$\frac{6+\lambda.14}{\lambda+1}-\frac{3\lambda^2}{(\lambda+1)^2}.6=6$$

$$\Rightarrow \frac{18\lambda^2}{(\lambda+1)^2}+6=6+\frac{8\lambda}{\lambda+1}$$

$$\Rightarrow 10\lambda=8$$

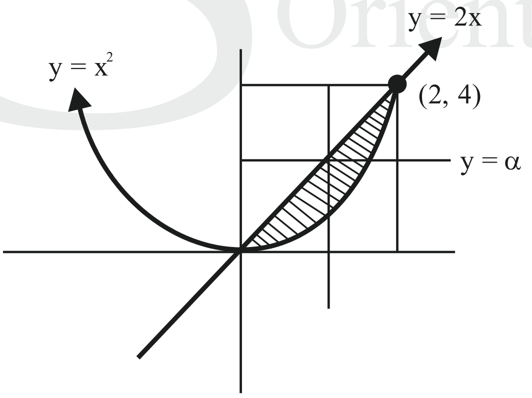
$$\Rightarrow \lambda=0.8$$

13. If $y=\alpha$ bisects the area bounded by region givenby $x^2\le y\le 2x$ then :

(a) $8\alpha^{3/2}-3\alpha^2=8$ (b) $8\alpha^{3/2}-3\alpha^2=4$ (c) $4\alpha^{3/2}-3\alpha^2=8$ (d) $4\alpha^{3/2}+3\alpha^2=8$

Ans. (a)

Sol :



$$\int_0^4\left(\sqrt{y}-\frac{y}{2}\right)dy=2\int_0^\alpha\left(\sqrt{y}-\frac{y}{2}\right)dy$$

$$\Rightarrow \frac{16}{3}-4=2\left(\frac{2}{3}\alpha^{3/2}-\frac{\alpha^2}{4}\right)$$

$$\Rightarrow \frac{4}{3}=2\left(\frac{2}{3}\alpha^{3/2}-\frac{\alpha^2}{4}\right)$$

$$\Rightarrow 8\alpha^{3/2}-3\alpha^2=8$$

14. If $\sin^4\theta+\cos^4\theta+\lambda=0$ has a real solution then range of λ is

(a) $[-1,1]$ (b) $\left[-1,\frac{-1}{2}\right]$ (c) $\left[\frac{1}{2},1\right]$ (d) $\left[-\frac{1}{2},\frac{1}{2}\right]$

Ans. (b)

Sol : $-\lambda=\sin^4\theta+\cos^4\theta$

$=\left(\sin^2\theta+\cos^2\theta\right)^2-2\sin^2\theta\cos^2\theta$

$=1-\frac{4\sin^2\theta\cos^2\theta}{2}$

$=1-\frac{\sin^2 2\theta}{2}$

$=\lambda=\frac{\sin^2 2\theta}{2}-1$

$\lambda\in\left[-1,-\frac{1}{2}\right]$

15. If $y=\sum_{k=1}^{6}K\cos^{-1}\left(\frac{3}{5}\cos kx-\frac{4}{5}\sin kx\right)$ then $\frac{dy}{dx}=?$

(a) 92 (b) 91 (c) 90 (d) 89

Ans. (b)

Sol : $y=\sum_{k=1}^{6}K\cos^{-1}\left(\cos kx.\cos\alpha-\sin kx.\sin\alpha\right)$

$=\sum k.\cos^{-1}\cos\left(kx+\alpha\right)$

$=\sum K\left(kx+\alpha\right)=\sum\left(k^2x+k\alpha\right)$

$=\frac{dy}{dx}=\sum_{k=1}^{6}k^2=\frac{6(7)(13)}{6}=91$

16. Let $a_1, a_2, ...a_{11}$ are in increasing A.P. and if variance of these number is 90 then value of common differentiation of A.P. is

Ans : 03.00

Sol : Given $a_1, a_2, ...a_{11}$, are in A.P.

$\therefore$ variance of $(a_1, a_2, ...a_{11})=90$

$\Rightarrow \frac{\sum_{i=1}^{11}a_i^2}{11}-\left(\frac{\sum_{i=1}^{11}a_i}{11}\right)^2=90$

$\Rightarrow \frac{d^2}{10}.\frac{10.11.21}{6}-d^2\frac{55}{11}.\frac{55}{11}=90$

$\Rightarrow 35d^2-25d^2=90$

$\Rightarrow d^2=9$

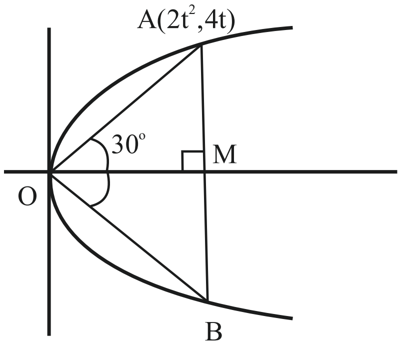
$\Rightarrow$ d = 3

17. An equilateral triangle is inscribed in parabola $y^2 = 8x$ whose one vertex coincide with vertex of parabola. Find area of triangle.

(a) $196\sqrt{3}$ (b) $194\sqrt{3}$ (c) $192\sqrt{3}$ (d) $190\sqrt{3}$

Ans. (c)

Sol :



$y^2 = 8x$, a = 2

A $\equiv (2t^2, 2)$ (b)t $\equiv (2t^2, 4t)$

$\tan 30^0 = \frac{4t}{2t^2} = \frac{2}{t} = \frac{1}{\sqrt{3}}$

$t = 2\sqrt{3}$

Area of ΔOAB = 2. ΔOMA = $2.\frac{1}{2}(2t^2)(4t) = 8t^3 = -8\left(2\sqrt{3}\right)^3 = 192\sqrt{3}$

18. If $f(x) = \frac{\ln(1+x)}{x}, x \in (-1, \infty)$ and f(0) = 1 then f(x) is

(a) decreasing in (-1, 0) and increasing in $(0, \infty)$

(b) always increasing

(c) always decreasing

(d) increasing in (-1, 0) and decreasing in $(0, \infty)$

Ans. (c)

Sol : $F'(x) = \frac{\frac{x}{1+x} - \ln(1+x)}{x^2} = \frac{x - (1+x)\ln(1+x)}{x^2(1+x)}$

Let g(x) = x - (1 + x) ln(1 + x)

$\Rightarrow$ g'(x) = 1 - 1 - ln(1 + x)

$= -\ln(1+x) \Rightarrow g'(x) = \begin{cases} > 0 & \forall x \in (-1, 0) \\ < 0 & \forall x \in (0, \infty) \end{cases}$

g_{max} at x = 0 $\Rightarrow$ g(0) = 0

g(x) < 0 $\forall x \in (-1, \infty)$ $\Rightarrow$ $f'(x) < 0 \forall x \in (-1, \infty)$

f(x) decreasing $\forall x \in (-1, \infty)$

19. If $y = (1 + x)^{2y} + \cos^2(\sin^{-1}x)$ be a curve then find equation of normal at x = 0.

(a) x + 4y = 8 (b) x + 4y = 2 (c) 2x + y = 2 (d) 2x - y = 2

Ans. (a)

Sol : $y = (1 + x)^{2y} + \cos^2(\sin^{-1}x)$

x = 0, y = 2

$$y = e^{2yl(1+x)} + \left(1 - x^2\right)$$

$$\frac{dy}{dx} = e^{2y\ln(1+x)}\left\{\frac{2y}{1+x} + \ln(1+x).2y'\right\} - 2x$$

$$y' = \left(\frac{2\times 2}{1+0} + 0\right)$$

y' = 4

y - 2 = -1/4 (x - 0)

x + 4y = 8

20. Which of the following is a tautology

(a) $\sim p \wedge (p \vee q) \rightarrow q$ (b) $\sim p \vee (p \vee q) \rightarrow q$

(c) $\sim p \vee (p \wedge q) \rightarrow q$ (d) None of these

Ans. (a)

Sol : (i) $\sim p \vee (p \vee q) \rightarrow q$

$(\sim p \wedge p) \vee (\sim p \wedge q) \rightarrow q$

$C \vee (\sim p \wedge q) \rightarrow q$

$(\sim p \wedge q) \rightarrow q$

$\sim(\sim p \wedge q) \vee q$

$= (p \vee \sim q) \vee q = p \vee t = t$

(ii) $\sim p \vee (p \vee q) \rightarrow q$

$(\sim p \vee q) \vee q \rightarrow q$

$t \vee q \rightarrow q$

$t \rightarrow q$

(iii) $\sim p \vee (p \wedge q) \rightarrow q$

$\Rightarrow (\sim p \vee q) \wedge (\sim p \vee q) \rightarrow q$

$t \wedge (\sim p \vee q) \rightarrow q$

$\Rightarrow \sim p \vee q \rightarrow q$

$\sim(\sim p \vee q) \vee q$

$(p \wedge \sim q) \vee q$

$(q \vee p) \wedge (q \vee \sim q) = q \vee p$

JEE MAIN 2020
PHASE 1

gravity
Orienting Intelligence

JEE Main - 2020 | Best Result in U.P.

Aditya Pandey
Percentile
99.936
City Topper
Application No. 200310320565
DOB - 23-12-2002

65 Students Above 99 Percentile
145 Students Above 98 Percentile
208 Students Above 97 Percentile

Abdullah (99.92)	Umayr (99.92)	Shivang Pandey (99.91)	Vinod Meena (99.89)	Jnaneswar Rao (99.89)	Pranav Rastogi (99.88)	Shiva Kumar (99.87)	Akshay Verma (99.86)

Neeraj Gartia (99.86)	Ayush Kumar (99.85)	Krishna Bhardwaj (99.85)	Ganesh (99.78)	Prashant Singh (99.74)	Adarsh Goyal (99.71)	Naman Dhingra (99.69)	Faraz Siddiqui (99.68)

Abhishek Kumar (99.66)	Avinesh Pratap (99.66)	Ajit Kumar (99.59)	Gitartha Bharadwaj (99.58)	Sameed (99.58)	Sameer (99.56)	Abhishek Kumar (99.54)	Ayush Jaiswal (99.52)

Sanjay Pratap (99.52)	Suryanshu Kumar (99.52)	Mihir Agarwal (99.50)	Mohil (99.49)	Brijesh Kumar (99.48)	Harshit Mishra (99.47)	Harsh Ghandwani (99.46)	Abdul Nasir (99.43)

Vinay Kumar (99.42)	Rustam Naryaan (99.42)	Tejeswar Reddy (99.39)	Pruthvi Raj (99.39)	Debdut Saini (99.38)	G Sai Kiran (99.37)	Sahil Kumar (99.32)	Pankaj (99.31)

Satyam Agarwal (99.31)	Divyanshu Yadav (99.28)	Piyush Tiwari (99.26)	Alisha Verma (99.23)	Avinash Kumar (99.19)	Amisha Verma (99.15)	Chetan Singh (99.13)	Ritik umar (99.13)

Shubham Kumar (99.10)	Randheer (99.08)	Aryansh Tripathi (99.06)	Saurabh Maurya (99.03)	Mohammad Affan (99.03)	Pradumna Awasthi (99.02)	Harsh (99.02)	Abhay (99.01)

SCHOOL INTEGRATED PROGRAM (SIP)

Tradition of Gravity Continues,
Once Again Historical Result,
100% Students Cracked
JEE Main
(Based on Last Yr Cut off)

2020

80 Out of 80
Cracked JEE Main

We had three Batches
of 55, 15 and 10.

Many Top Ranks are
from these Batches

2019

79 Out of 80 in JEE Main	50 Out of 79 in JEE Adv.

2018

83 Out of 85 in JEE Main	62 Out of 83 in JEE Adv.

2017

80 Out of 85 in JEE Main	63 Out of 80 in JEE Adv.

2016

39 Out of 40 in JEE Main	31 Out of 39 in JEE Adv.

Selections Engineering 2019

gravity
Orienting Intelligence


Tarun

194 AIR (General)


Aniket Agarwal

337 AIR (General)


Shubh Sahu

494 AIR (General)


Shlok Nemani

497 AIR (General)

50 out of 79 Cracked JEE Advanced from SIP (School Integrated Program)

4 Ranks under 500 (General Category) | 2 Ranks under 10 (Reserved Category)

126 Selections in JEE Advanced | 61 Students above 99 Percentile in JEE Main 2019


Sanjana

AIR - 3*


Akash

AIR - 4*


Priyanka

AIR - 68*


Bibek Lakra

AIR - 150*


Neha Raj

AIR - 177*


Arindam

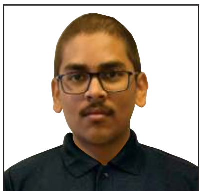
AIR - 809 (General EWS)


Priyam

AIR - 1378 (General)


Mihir Chawla

AIR - 2237 (General)


Madhur Kumar

AIR - 2382 (General)


Manish Kumar

AIR - 2388 (General)


Saumya Raj

AIR - 2656 (General)


Raghav

AIR - 2659 (General)


Ritveek

AIR - 2709 (General)


Vanshaj

AIR - 2787 (General)


Subir Gupta

AIR - 2881 (General)


Aryan Rastogi

AIR - 3167 (General)


Devansh

AIR - 3600 (General)


Abhisht Bose

AIR - 3784 (General)