JEE Main (Phase-II) 2020

Memory Based Questions & Solutions

SUBJECT

MATHEMATICS

Date: 03 September, 2020 (Shift-1)

Time: 9 AM to 12 PM

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1. A circle touch the line $x = 3$ and $y = 2$ and a line $x + y = 2$ passes through its centre of circle then find the diameter of circle.

(1) 2 (2) 3 (3) 4 (4) 5

Ans. (2)

Sol. Centre will lie on bisector of $x = 3$ and $y = 2$

$$\frac{x-3}{1} = \frac{y-2}{1}$$

$\Rightarrow$ $x - 3 = y - 2$ $\quad$ $x - 3 = 2 - y$

$\Rightarrow$ $x - y = 1$ $\quad$ $x + y = 5$ (not possible)

Solve $x + y = 2$

$x - y = 1$

$$x = \frac{3}{2},\ y = \frac{1}{2}$$

$$\text{Radius} = \left|\frac{3}{2} - 3\right| = \frac{3}{2}$$

Diameter = 3

2. In the Matrix $A = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$ and $A^4 = \begin{bmatrix} 109 & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ Find the value of a_{22}

(1) 9 (2) 10 (3) 11 (4) 12

Ans. (2)

Sol. $A^2 = \begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$

$$= \begin{bmatrix} x^2+1 & x \\ x & 1 \end{bmatrix}\begin{bmatrix} x^2+1 & x \\ x & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (x^2+1)^2 + x^2 & x(x^2+1) + x \\ x(x^2+1) + x & x^2+1 \end{bmatrix}$$

$(x^2+1)^2 + x^2 = 109$ $\quad$ Let $x^2 = t$

$t^2 + 2t + 1 + t = 109$

$\Rightarrow t^2 + 3t - 108 = 0$ $\quad$ $\Rightarrow (t+12)(t-9) = 0$

$t = 9$ $\quad$ $x^2 = 9$

Requires Ans. $= x^2 + 1 = 10$

3. A die is rolled twice and the sum is noted be a multiple of four. Find probability of getting a score of four atleast once

(1) 1/9 (2) 1/2 (3) 1/3 (4) 1/4

Ans. (1)

Sol. Sample space

= {1, 3), (2, 2), (3, 1), (2, 6), (3, 5), (4, 4), (5, 3), (6, 2), (6, 6)}

n(s) = 9

4 is occuring in only one of these 9 case (4, 4)

$$\text{So } P(E) = \frac{n(E)}{n(S)} = \frac{1}{9}$$

4. $\left(2^{\frac{1}{2}}+5^{\frac{1}{8}}\right)^{n}$ has 33 integral terms find least value of n

(1) 256 (2) 257 (3) 258 (4) 259

Ans. (1)

Sol. $T_{r+1}={}^{n}C_{r}\left(2^{1/2}\right)^{n-r}\left(5^{1/8}\right)^{r}$

$={}^{n}C_{r}\ 2^{\frac{n-r}{2}}\ 5^{\frac{r}{8}}$

L.C.M. of (2, 8)= 8

So n should be = (33 - 1) x 8

= 32 x 8

= 256

5. $I=\int_{-\pi}^{\pi}|\pi-|x||dx$ Find the value of I.

(1) π^2 (2) π^3 (3) $2\pi^2$ (4) 0

Ans. (1)

Sol. $I=2\int_{0}^{\pi}|\pi-|x||dx$

$=2\int_{0}^{\pi}|\pi-x|dx$

$=2\int_{0}^{\pi}(\pi-x)dx=2\left[\pi x-\frac{x^2}{2}\right]_{0}^{\pi}$

$=2\left[\pi^2-\frac{\pi^2}{2}\right]=\pi^2$

6. If the first term of an A.P. is 3 and sum of first 25 terms is equal to sum of next 15 terms. Then find common difference of the A.P.

(1) 1/6 (2)1/31 (3) 30 (4) 31

Ans. (1)

Sol. $\frac{25}{2}[a_1+a_{25}]=\frac{15}{2}[a_{26}+a_{40}]$

5[a + a + 24d] = 3[a + 25d + a + 39d]

10a + 120d = 6a + 3 x 64d

4a = 192d - 120d

4a = 72d

$\Rightarrow d=\frac{a}{18}=\frac{3}{18}=\frac{1}{6}$

7 Find the value of K $\lim_{x\to 0}\frac{1}{x^8}\left(1-\cos\frac{x^2}{2}-\cos\frac{x^2}{4}+\cos\frac{x^2}{2}\cos\frac{x^2}{4}\right)=2^{-k}$

(1) 1/4 (2)1/5 (3) 6 (4) 5

Ans. (3)

Sol. $= \lim_{x\to 0} \frac{1}{x^8}\left(1-\cos\frac{x^2}{2}\right)\left(1-\cos\frac{x^2}{4}\right)$

$= \lim_{x\to 0} \frac{\sin^2\left(\frac{x^2}{2}\right)}{x^4}.\frac{\sin^2\left(\frac{x^2}{4}\right)}{x^4}$

$= \frac{1}{4}x\frac{1}{16} = \frac{1}{64} = 2^{-6} = 2^{-K}$

$K = 6$

8. $L = \lim_{x\to 0}\left|\frac{1-x+|x|}{\lambda - x + [x]}\right|$ $\lambda \in R - \{0,1\}$ where [.] is Greatest integer function Find L.

(1) 2 (2) 3 (3) 4 (4) 5

Ans. (1)

Sol. $= \lim_{x\to 0^+}\left|\frac{1-x+x}{\lambda - x + 0}\right| = \frac{1}{|\lambda|}$ $= \lim_{x\to 0^-}\left|\frac{1-x+|x|}{\lambda - x + [x]}\right|$

$= \lim_{x\to 0^-}\left|\frac{1-2x}{\lambda - x - 1}\right| = \frac{1}{|\lambda - 1|}$ $\Rightarrow |\lambda| = |\lambda - 1| \Rightarrow \lambda = \frac{1}{2}$

$\therefore\ L = \frac{1}{1/2} = 2$

9. Find the value of $0.16^{\log_{2.5}\left(\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}.....\infty\right)}$

(1) 2 (2) 3 (3) 4 (4) 5

Ans. (3)

Sol. $= (0.16)^{\log_{5/2}^{(1/2)}}$

$= \left(\frac{1}{2}\right)^{\log_{5/2}^{(0.16)}} = \left(\frac{1}{2}\right)^{\log_{(2/5)}^{(2/5)^2}}$

$= \left(\frac{1}{2}\right)^{-2} = 2^2 = 4$

10. $\left(\frac{1+i}{1-i}\right)^{\frac{m}{2}} = \left(\frac{1+i}{i-1}\right)^{\frac{n}{3}} = 1$ find GCD of m & n

(1) 4 (2) 8 (3) 12 (4) 24

Ans. (4)

Sol. $\left(\frac{1+i}{1-i}\right)^{\frac{m}{2}} = \left(\frac{(1+i)(1+i)}{2}\right)^{\frac{m}{2}} = (i)^{\frac{m}{2}} \Rightarrow m = 8$

$\left(\frac{i+1}{i-1}\right)^{\frac{n}{3}} = \left(\frac{(i+1)(i+1)}{-2}\right)^{\frac{n}{3}} = (-i)^{\frac{n}{3}} \Rightarrow n = 12$

$GCD\ (m,n) = 4$

11. A bag contains 6 red and 10 green balls, 3 balls are drawn from it one by one without replacement. If the third ball drawn is red then the probability that first two balls are green is:

(1) 3/7 (2) 9/149 (3) 9/56 (4) 3/8

Ans. (1)

Sol. $\text{Probability} = \dfrac{P(g_1g_2R_3)}{P(R_1R_2R_3)+P(R_1g_2R_3)+P(g_1R_2R_3)+P(g_1g_2R_3)}$

$$= \frac{10\cdot 9\cdot 6}{6\cdot 5\cdot 4+6\cdot 10\cdot 5+10\cdot 6\cdot 5+10\cdot 9\cdot 6}$$

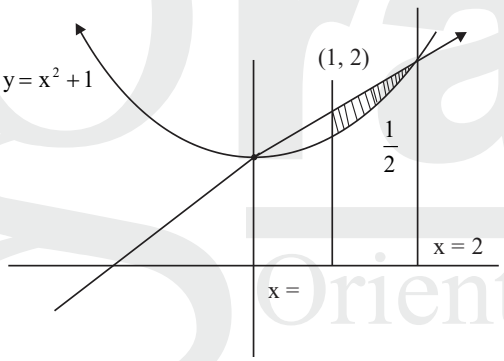
$$= \frac{90}{20+50+50+90} = \frac{9}{21} = \frac{3}{7}$$

12. Area enclosed by $0 \le y \le x^2+1$, $0 \le y \le x+1$, $\frac{1}{2} \le x \le 2$ is :

(1) $\frac{1}{12}$ (2) $\frac{1}{6}$ (3) 1 (4) $\frac{1}{3}$

Ans. (1)

Sol.



$$A = \int_{1/2}^{1}\left[(x+1)-(x^2+1)\right]dx$$

$$\Rightarrow \int_{1/2}^{1}(x-x^2)\,dx$$

$$\Rightarrow \left[\frac{x^2}{2}-\frac{x^3}{3}\right]_{1/2}^{1} = \left(\frac{1}{2}-\frac{1}{3}\right)-\left(\frac{1}{8}-\frac{1}{24}\right) = \frac{1}{6}-\frac{1}{12} = \frac{1}{12}$$

13. The equation of curve satisfying differential equation $(1+y^2)(e^x+1)\,dy = e^x y^2\,dx$ and also passes through the point (0, 1) is

(1) $y^2+1 = y\ell n\left(\dfrac{1+e^x}{2}\right)$ (2) $y^2-1 = y\ell n\left(\dfrac{1+e^x}{2}\right)$

(3) $y+1 = y\ell n\left(\dfrac{1+e^x}{2}\right)$ (4) $2y^2+1 = y\ell n\left(\dfrac{1+e^x}{2}\right)$

Ans. (2)

Sol. Given equation is $\dfrac{1+y^2}{y^2}dy = \dfrac{e^x}{1+e^x}dx \Rightarrow \left(\dfrac{1}{y^2}+1\right)dy = \dfrac{e^x dx}{1+e^x} \Rightarrow -\dfrac{1}{y}+y = \ln\left(1+e^x\right)+C$

$\therefore$ passes through (0, 1)

$\therefore$ $-1+1=\ln 2+C \Rightarrow C=-\ln 2$

equation of curve $y-\frac{1}{y}=\ln\left(1+e^{x}\right)-\ln 2$

$\Rightarrow$ $y^2-1=y\ln\left(\frac{1+e^{x}}{2}\right)$

14. $2\pi-\left[\sin^{-1}\frac{4}{5}+\sin^{-1}\frac{5}{13}+\sin^{-1}\frac{16}{65}\right]=$

(1) $\frac{\pi}{2}$ (2) π (3) $\frac{3\pi}{2}$ (4) $-\frac{\pi}{2}$

Ans. (3)

Sol. $2\pi-\left[\tan^{-1}\frac{4}{3}+\tan^{-1}\frac{5}{12}+\tan^{-1}\frac{16}{63}\right]$

$\Rightarrow$ $2\pi-\left[\tan^{-1}\left(\frac{\frac{4}{3}+\frac{5}{12}}{1-\frac{4}{3}.\frac{5}{12}}\right)+\tan^{-1}\frac{16}{63}\right]$

$\Rightarrow$ $2\pi-\left[\tan^{-1}\frac{63}{16}+\cot^{-1}\frac{63}{16}\right]=2\pi-\frac{\pi}{2}=\frac{3\pi}{2}$

15. If $y^2+\ell n\cos^2 x=y$ then

(1) $|y''(0)|=2$ (2) $|y'(0)|+|y''(0)|=1$

(2) $|y'(0)|+|y''(0)|=3$ (3) None of these

Ans. (1)

Sol. $2y\,y'-2\tan x=y'$ ..... (1)

$2y\,y''+2(y')^2-2\sec^2 x=y''$ ..... (2)

when $x=0 \Rightarrow y=0, 1$

if $x=0$ and $y=0 \Rightarrow y'(0)=0 \Rightarrow y'(0)=0$

if $x=0$ and $y=1 \Rightarrow y'(0)=0$

equation (2) $2y''+2(y')^2-2\sec^2 x=y''$

at $x=0 \Rightarrow 2y\,y''-2=y'' \Rightarrow y''(2y-1)=2$

$\Rightarrow$ $y''(0)=\frac{2}{2y-1}\begin{cases}-2 & ; \ y=0\\ 2 & y=1\end{cases}$ $\Rightarrow |y''(0)|=2$

16. $S=(2\,{}^{1}p_0-3\,{}^{2}p_1+4\,{}^{3}p_2+\text{.....51 terms})+(1!-2!+3!-4!)+\text{.....51 terms})$, find S

(1) 1 + 51! (2) 1 + 52! (3) 1 + 50 (51!) (4) 1 + 51 (51!)

Ans. (2)

Sol. $S=(2\,{}^{1}p_0-3\,{}^{2}p_1+4\,{}^{3}p_2+\text{..... 51 terms})+(1!-2!+3!-4!+\text{..... 51 terms})$

$=2!-3!+4!-5!+\text{.....}52!+(1!-2!+3!-4!+\text{.....}+51!)=1+52!$

17. The preposition $p\Rightarrow(\sim(p\wedge\sim q))$ is equivalent to

(1) q (2) $\sim p\vee q$ (3) $p\vee\sim q$ (4) $\sim p\vee\sim q$

Ans. (2)

Sol. Given $\equiv p \Rightarrow (\sim (p \wedge \sim q))$

$\equiv \sim p \vee (\sim (p \wedge \sim q))$

$\equiv \sim p \vee (\sim p \vee q)$

$\equiv \sim p \vee q$

18. Let $\frac{x^2}{4}+\frac{y^2}{3}=1$ is an ellipse and a hyperbola which is confocal with ellipse such that its transverse excess is $\sqrt{2}$ then which of following point does not lies on hyperbola

(1) $\left(1,-\frac{1}{\sqrt{2}}\right)$ (2) $\left(-\sqrt{\frac{3}{2}},1\right)$ (3) $\left(\sqrt{\frac{3}{2}},\frac{1}{\sqrt{2}}\right)$ (4) None of these

Ans. (3)

Sol. ellipse $\frac{x^2}{4}+\frac{y}{3}=1$ $a_1=2b_1=\sqrt{3}$ $e_1=\sqrt{1-\frac{3}{4}}=\frac{1}{2}$

Focus $=(\pm 1,0)$

for hyperbola $a_2=\frac{1}{\sqrt{2}}$ and $a_2\, e_2=1 \Rightarrow e_2=\sqrt{2} \Rightarrow b_2=a_2=\frac{1}{\sqrt{2}}$

hyperbola $\frac{x^2}{a_2^2}-\frac{y^2}{b_2^2}=1$

$\Rightarrow x^2=y^2=\frac{1}{2}$

Gravity
Orienting Intelligence

JEE MAIN 2020
PHASE 1

JEE Main - 2020

Best Result in U.P.

Aditya Pandey

Percentile

99.936

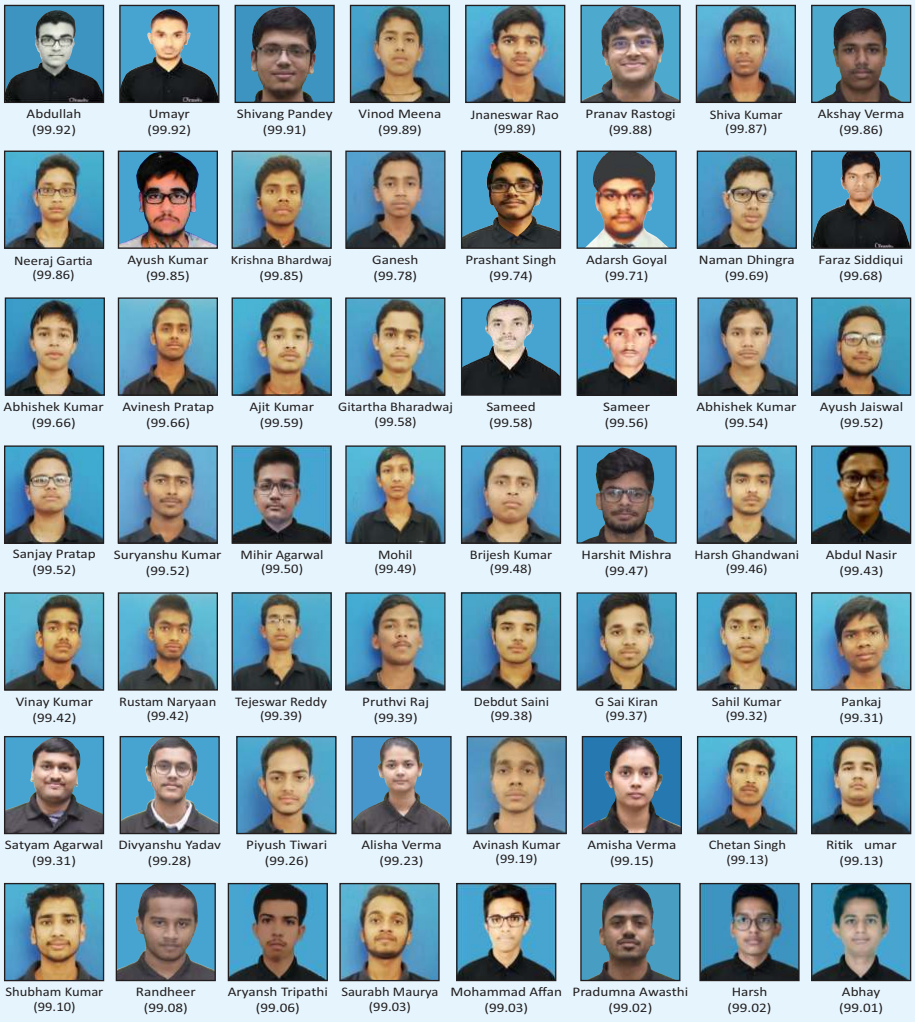
City Topper

Application No. 200310320565
DOB - 23-12-2002

65 Students Above 99 Percentile

145 Students Above 98 Percentile

208 Students Above 97 Percentile



SCHOOL INTEGRATED PROGRAM (SIP)

Tradition of Gravity Continues,
Once Again Historical Result,
100% Students Cracked
JEE Main
(Based on Last Yr Cut off)

2020

80 Out of 80
Cracked JEE Main

We had three Batches
of 55, 15 and 10.

Many Top Ranks are
from these Batches

2019

79 Out of 80 in JEE Main	50 Out of 79 in JEE Adv.

2018

83 Out of 85 in JEE Main	62 Out of 83 in JEE Adv.

2017

80 Out of 85 in JEE Main	63 Out of 80 in JEE Adv.

2016

39 Out of 40 in JEE Main	31 Out of 39 in JEE Adv.

Selections Engineering 2019

gravity
Orienting Intelligence

194
AIR
(General)

Tarun

337
AIR
(General)

Aniket Agarwal

494
AIR
(General)

Shubh Sahu

497
AIR
(General)

Shlok Nemani

50 out of 79 Cracked JEE Advanced from SIP (School Integrated Program)

4 Ranks under 500 (General Category) | 2 Ranks under 10 (Reserved Category)

126 Selections in JEE Advanced | 61 Students above 99 Percentile in JEE Main 2019

Sanjana

AIR - 3*

Akash

AIR - 4*

Priyanka

AIR - 68*

Bibek Lakra

AIR - 150*

Neha Raj

AIR - 177*

Arindam

AIR - 809
(General EWS)

Priyam

AIR - 1378
(General)

Mihir Chawla

AIR - 2237
(General)

Madhur Kumar

AIR - 2382
(General)

Manish Kumar

AIR - 2388
(General)

Saumya Raj

AIR - 2656
(General)

Raghav

AIR - 2659
(General)

Ritveek

AIR - 2709
(General)

Vanshaj

AIR - 2787
(General)

Subir Gupta

AIR - 2881
(General)

Aryan Rastogi

AIR - 3167
(General)

Devansh

AIR - 3600
(General)

Abhisht Bose

AIR - 3784
(General)