JEE Main (Phase-II) 2020

Memory Based Questions & Solutions

SUBJECT

MATHEMATICS

Date: 02 September, 2020 (Shift-1)

Time: 9 AM to 12 PM

1. The rank of the word MOTHER is
(a) 308 (b) 309
(c) 310 (d) 311

Ans. (b)

Sol. E.....→ $\lfloor 5$

H.....→ $\lfloor 5$

ME.....→ $\lfloor 4$

MH.....→ $\lfloor 4$

MOE.....→ $\lfloor 3$

MOH.....→ $\lfloor 3$

MOR.....→ $\lfloor 3$

MOTE.....→ $\lfloor 2$

MOTHER → $\lfloor 1$

So Rank

$= 2\lfloor 5 + 2\lfloor 4 + 3\lfloor 3 + \lfloor 2 + 1$

= 240 + 48 + 18 + 3 = 309

2. The area bounded by region $\frac{x^2}{4}+\frac{y^2}{9}=1$ & $\frac{|x|}{2}+\frac{|y|}{3}=1$ is
(a) 3(π–2) (b) 4(π–2)
(c) 5(π–2) (d) 6(π–2)

Ans. (d)

3. $\lim_{x\to 1}\frac{x+x^2+x^3+.....+x^n-n}{x-1}=820$

then n equals
(a) 40 (b) 41
(c) 59 (d) 60

Ans. (a)

Sol. Using L.H. rule

$\lim_{x\to 1}\frac{1+2x+3x^2+.....+nx^{n-1}}{1}=820$

1 + 2 + 3 +.....+ n = 820

n(n + 1)/2 = 820

n = 40

4. Three term of G.P. Has sum S & product 27 than the range of S is
(a) $[-\infty,-3]\cup[9,\infty)$
(b) $[-\infty,-4]\cup[9,\infty)$
(c) $[-\infty,-5]\cup[10,\infty)$
(d) $[-\infty,-6]\cup[11,\infty)$

Ans. (a)

Sol. Let the terms are a, ar, ar^2

$S = a(1 + r + r^2)$

$P = (a \times r)^3 = 27 \Rightarrow ar = 3$

$S = \frac{3}{r}(1 + r + r^2)$

$=3\left(r+\frac{1}{r}+1\right)$

Now, $r+\frac{1}{r}\ge 2$ or $r+\frac{1}{r}\le -2$

$S\ge 9$ or $S\le -3$

$[-\infty,-3]\cup[9,\infty)$

5. $x^2 + y^2 - 2x - 4y + 4 = 0$ is circle whose tangent is $3x + 4y = k$ then number of integral values of k are
(a) 3 (b) 2 (c) 4 (d) 1

Ans. (b)

Sol. Since the straight line is tangent to the circle so the distance of centre from the line is equal to radius. therefore $\left|\frac{3+8-k}{5}\right|=1$

$|11-k|=5$

$k=16$ or 6

therefore k has 2 integral values

6. Box-1 contains the number from the 1 to 30 and Box-2 contains the number from 31 to 50. A number is selected at random, and was found to be non prime then the probability that number is selected from Box -1, is
(a) $\frac{8}{17}$ (b) $\frac{5}{17}$
(c) $\frac{6}{17}$ (d) None of these

Ans. (a)

Sol. Box-1 has 20 non primes while the box 2 has 15 non primes. Therefore using Baye theorem

probability $p=\frac{\left(\frac{20}{30}\right)}{\left(\frac{20}{30}\right)+\left(\frac{15}{20}\right)}$

$p=\frac{8}{17}$

7. (2, 2, 1) & (2, 1, 2) are two points in a plane which is parallel to the line 2x = 3y, z = 1. Then which of the following point lie in the plane.
(a) (2, 1, –1) (b) (3, –2, 0)
(c) (4, 7, –8) (d) (–1, 1, 0).

Ans (d)

Sol $a(x-2)+b(y-2)+c(z-1)=0$.....(1)
$a(2-2)+b(1-2)+c(2-1)=0$
$0\times a-b+c=0$.....(2)
$3a+2b+0\times c=0$.....(3)

$$\begin{vmatrix} x-2 & y-2 & z-1 \\ 0 & -1 & 1 \\ 3 & 2 & 0 \end{vmatrix}=0$$

$(x-2)(-2)-(y-2)(-3)+(z-1)(3)=0$
$-2x+3y+3z=5$
Now, we can easily check that
(–1, 1, 0) lies on the plane

8. If $ax^2+bx+6=0$ does not have two distinct real roots, $a\in R$, $b\in R$, then the least value of $(3a+b)$ is
(a) 4 (b) –1
(c) 1 (d) –2

Ans. (d)

Sol. $ax^2+bx+6=0$
$D\le 0$
$\Rightarrow ax^2+bx+6\ge 0$
put x = 3
$9a+3b+6\ge 0$
$\Rightarrow 3a+b\ge -2$

9. The length and foot of perpendicular from (1, 3/2, 2) to the plane 2x – 2y + 4z + 5 = 0 are
(a) $\sqrt{6},\left(0,\frac{5}{2},0\right)$ (b) $\sqrt{5},\left(0,\frac{5}{2},0\right)$
(c) $\sqrt{5},\left(1,\frac{5}{2},1\right)$ (d) $\sqrt{7},\left(1,\frac{5}{2},1\right)$

Ans. (a)

Sol. $p\left(1,\frac{3}{2},2\right), 2x-2y+4z+5=0$
Foot of perpendicular

$$\frac{x-1}{2}=\frac{y\frac{3}{2}}{-2}=\frac{z-2}{4}=\frac{-(2-3+8+5)}{4+4+16}$$

$$\Rightarrow \frac{x-1}{2}=\frac{y-\frac{3}{2}}{-2}=\frac{z-2}{4}-\frac{12}{24}=\frac{-1}{2}$$

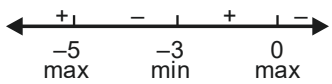
$$\Rightarrow x=1-1, y=\frac{3}{2}+1, z=2-2$$

$$\Rightarrow x=0, y=\frac{5}{2}, z=0$$

10. The function $f(x)=-(3/4)x^4-8x^3-(45/2)x^2+105$ has
(a) local maxima at x = 0, x = –5 & local minima at x = – 3
(b) local maxima at x = 1, x = –5 & local minima at x = – 3
(c) local maxima at x = 2, x = –5 & local minima at x = – 3
(d) local maxima at x = 3, x = –5 & local minima at x = – 3

Ans. (a)

Sol. $f'(x)=13x^3-24x^2-45x+0$
$=-3x(x+3)(x+5)$



11. The contrapositive of the statement "If I reach the station in time then I will catch the train"
(a) If I will not catch the train then I did not reach the station in time
(b) I catch the train if I reached the station in time
(c) If I reach the station in time, I will not catch the train.
(d) None of these

Ans. (a)

Sol. If I will not catch the train then I did not reach the station in time

12. $\left(\frac{1+\sin\frac{2\pi}{9}+i\cos\frac{2\pi}{9}}{1+\sin\frac{2\pi}{9}-i\cos\frac{2\pi}{9}}\right)^3$ equals

(a) $-\frac{\sqrt{3}}{2}+i\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}+\frac{i}{2}$
(c) $-\frac{\sqrt{3}}{2}-i\frac{1}{2}$ (d) $\frac{\sqrt{3}}{2}-i\frac{1}{2}$

Ans. (a)

Sol. $$\left(\frac{1+\cos\frac{5\pi}{18}+i\sin\frac{5\pi}{18}}{1+\cos\frac{5\pi}{18}-i\sin\frac{5\pi}{18}}\right)^3$$

$$=\left(\frac{2\cos^2\frac{5\pi}{36}+i2\sin\frac{5\pi}{36}.\cos\frac{5\pi}{36}}{2\cos^2\frac{5\pi}{36}-i2\sin\frac{5\pi}{36}.\cos\frac{5\pi}{36}}\right)^3$$

$$=\left(\frac{\cos\frac{5\pi}{36}+i\sin\frac{5\pi}{36}}{\cos\frac{5\pi}{36}-i\sin\frac{5\pi}{36}}\right)^3$$

$$=\left(\cos\frac{5\pi}{36}+i\sin\frac{5\pi}{36}\right)^6$$

$$=\cos\left(6\times\frac{5\pi}{36}\right)+i\sin\left(6\times\frac{5\pi}{36}\right)$$

$$=\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}$$

$$\Rightarrow\quad -\frac{\sqrt{3}}{2}+i\frac{1}{2}$$

13. If $\frac{(1+\sin x)}{y+1}\frac{dy}{dx}=-\cos x$ such that y(0) = 1 and y(π) = a & $\left(\frac{dy}{dx}\right)_{(x=\pi)}=b$, then (a, b) =

(a) (1, 2) (b) (1, –2)
(c) (–1, 2) (d) None of these

Ans. (a)

So. $$\frac{dy}{1+y}=\frac{-\cos x}{1+\sin x}dx$$

$$\ell n(1+y)=-\ell n(1+\sin x)+\ell nc$$

$$(1+y)(1+\sin x)=c$$

$$2.1=c\quad\Rightarrow\quad c=2$$

$$1+y=\frac{2}{1+\sin x}\Rightarrow\quad y=\frac{2}{1+\sin x}-1$$

$$y(\pi)=2-1=1$$

$$\frac{dy}{dx}=\frac{-2}{(1+\sin x)^2}.\cos x=2 \text{ at } x=\pi$$

$$(a,b)=(1,2)$$

14. If $\vec{a},\vec{b},\vec{c}$ are three unit vectors such that $|\vec{a}-\vec{b}|^2+|\vec{a}-\vec{c}|^2=8$ then find the value of $|\vec{a}+2\vec{b}|^2+|\vec{a}+2\vec{c}|^2$

(a) 1 (b) 2
(c) 4 (d) None of these

Ans. (b)

Sol. $|\vec{a}-\vec{b}|^2+|\vec{a}-\vec{c}|^2=8$

$$(\vec{a}-\vec{b})\cdot(\vec{a}-\vec{b})+(\vec{a}-\vec{c})\cdot(\vec{a}-\vec{c})=8$$

$$1+1-2\vec{a}\cdot\vec{b}+1+1-2\vec{a}\cdot\vec{c}=8$$

$$\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}=-2$$

Now $|\vec{a}+2\vec{b}|^2+|\vec{a}+2\vec{c}|^2$

$$=\left(\vec{a}+2\vec{b}\right)\cdot\left(\vec{a}+2\vec{b}\right)+\left(\vec{a}+2\vec{c}\right)\cdot\left(\vec{a}+2\vec{c}\right)$$

$$=1+4+4\,\vec{a}\cdot\vec{b}+1+4+4\,\vec{a}\cdot\vec{c}$$

$$=10+4(-2)=2$$

15. Evaluate $\int_0^2||x-1|-x|\,dx$

(a) 1/2 (b) 3/2
(c) 5/2 (d) 2

Ans. (d)

Sol. $\int_0^2|x-1|-x|\,dx$

$$=\int_0^1|1-x-x|\,dx+\int_1^2|x-1-x|\,dx$$

$$=\int_0^{1/2}(1-2x)dx+\int_{1/2}^1(2x-1)dx+\int_1^2 dx$$

$$\left[x-x^2\right]_0^{\frac{1}{2}}+\left[x^2-x\right]_{\frac{1}{2}}^1+[x]_1^2=3/2$$

16. Solution of equation

$\sin^{-1}6x+\sin^{-1}6\sqrt{3}x=-\frac{\pi}{2}$ is

(a) 1/12 (b) –1/12
(c) ±1/12 (d) None of these

Ans. (b)

Sol. $\sin^{-1}6x=\sin^{-1}(-1)-\sin^{-1}(6\sqrt{3}x)$

$$\Rightarrow\quad \sin^{-1}6x=\sin^{-1}\left(-1\sqrt{1-108x^2}-6\sqrt{3}x\sqrt{1-1}\right)$$

$$\Rightarrow\quad 36x^2=1-108x^2\quad\Rightarrow\quad 144x^2=1$$

$$\Rightarrow x=-\frac{1}{12}$$

17. P(x) is a polynomial of degree 3 which have maximum value 8 at x = 1, min value 6 at x = 2 then find P(0)

(a) 0 (b) 1
(c) –2 (d) None of these

Ans. (c)

Sol. Clearly $P'(x) = \lambda(x-1)(x-2)$ where $\lambda > 0$

$$P(x) = \lambda\left[\frac{x^3}{3} - \frac{3x^2}{2} + 2x\right] + C$$

$$\text{given } P(1) = 8 \Rightarrow \lambda\left(\frac{1}{3} - \frac{3}{2} + 2\right) + C = 8$$

$$\Rightarrow \frac{5\lambda}{6} + C = 8$$

$$\text{Also} P(2) = 6 \Rightarrow 6 \Rightarrow \lambda\left(\frac{8}{3} - 6 + 4\right) + C = 6$$

$$\Rightarrow \frac{2}{3}\lambda + C = 6$$

$$\text{By(i) and (ii)} \Rightarrow C = -2$$

$$\Rightarrow P(0) = -2$$

18. $\frac{z-\alpha}{z+\alpha}$ is purely imaginary and |z| = 82 then |α| is

(a) 80 (b) 82
(c) $\sqrt{82}$ (d) $2\sqrt{82}$

Ans. (b)

Sol. $$\frac{z-\alpha}{z+\alpha} + \frac{\bar{z}-\bar{\alpha}}{\bar{z}+\bar{\alpha}} = 0$$

$$2z\bar{z} - \alpha\bar{z} + \bar{\alpha}z - z\bar{\alpha} + \alpha\bar{z} - 2\alpha\bar{\alpha} = 0$$

$$1|z|^2 = 2|\alpha|^2$$

$$|\alpha| = 82$$

20. $A = \{x \in N : 1 \le x \le 17\}$

$B = \{ax + b : x \in A\} a > 0$

Variance of set B is 2016 and mean is 17 find a + b

(a) –7 (b) 7
(c) 6 (d) –6

Ans. (a)

Sol. $$B(\bar{x}) = a\bar{x} + b = \frac{a(1+2+3+....+17)}{17} + b = 17$$

$$\frac{a.(17.18)}{17.2} + b = 27$$

$$9a + b = 17 \quad .....(i)$$

$$\sigma A^2 = \frac{\sum x^2}{n} = \left(\frac{\sum x}{n}\right)^2$$

$$= \frac{1^2 + 2^2 + ... + 17^2}{17} - \left(\frac{1+2+....17}{17}\right)^2$$

$$= \frac{17.18.35}{6.17} - \left(\frac{17.18}{2.17}\right)^2$$

$$= 105 - 81 = 24$$

$$\therefore \sigma^2 B = a^2 \sigma A^2 = a^2.24 = 216$$

$$a^2 = \frac{216}{24} = 9$$

$$a = 3 \quad \therefore b = 17 - 27$$

$$\therefore b = 17 - 27$$

$$b = -10$$

$$\therefore a + b = -7$$

21. If equation $6x^2 + 5x - 2 = 0$ has roots α, β and $S_n = \alpha^n + \beta^n$. Then select the correct option

(a) $6S_6 + 5S_5 = 2S_4$ (b) $6S_4 + 2S_5 = 5S_6$
(c) $6S_2 + 5S_6 = 6S_4$ (d) $6S_2 + 5S_4 = 2S_6$

Ans. (a)

Sol. $6a^2 + 5a = 2$

$6S_6 + 5S_5 = \alpha^4(6\alpha^2 + 5\alpha) + \beta^4(6\beta^2 + 5\beta)$

$= 2(\alpha^4 + \beta^4) = 2S_4$

Gravity
Orienting Intelligence

JEE Main - 2020

Best Result in U.P.

Aditya Pandey

Percentile

99.936

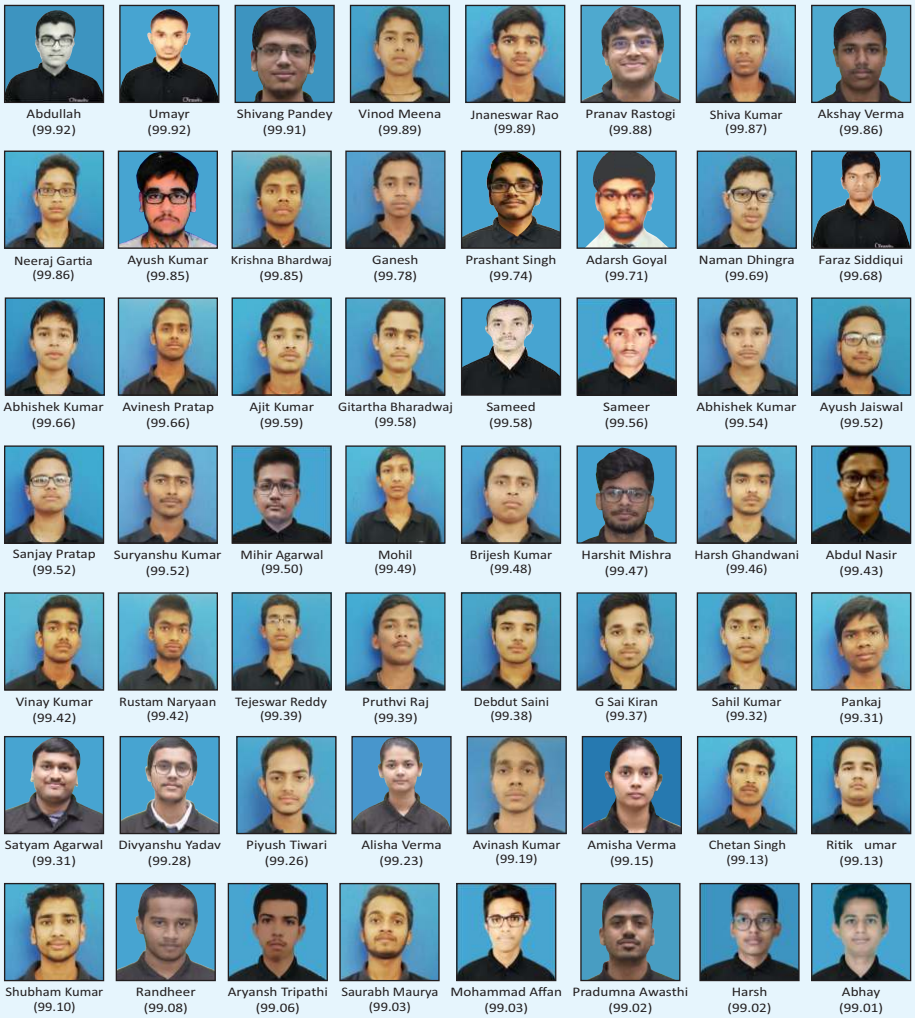
City Topper

Application No. 200310320565
DOB - 23-12-2002

65 Students Above 99 Percentile

145 Students Above 98 Percentile

208 Students Above 97 Percentile



Abdullah (99.92)	Umayr (99.92)	Shivang Pandey (99.91)	Vinod Meena (99.89)	Jnaneswar Rao (99.89)	Pranav Rastogi (99.88)	Shiva Kumar (99.87)	Akshay Verma (99.86)
Neeraj Gartia (99.86)	Ayush Kumar (99.85)	Krishna Bhardwaj (99.85)	Ganesh (99.78)	Prashant Singh (99.74)	Adarsh Goyal (99.71)	Naman Dhingra (99.69)	Faraz Siddiqui (99.68)
Abhishek Kumar (99.66)	Avinesh Pratap (99.66)	Ajit Kumar (99.59)	Gitartha Bharadwaj (99.58)	Sameed (99.58)	Sameer (99.56)	Abhishek Kumar (99.54)	Ayush Jaiswal (99.52)
Sanjay Pratap (99.52)	Suryanshu Kumar (99.52)	Mihir Agarwal (99.50)	Mohil (99.49)	Brijesh Kumar (99.48)	Harshit Mishra (99.47)	Harsh Ghandwani (99.46)	Abdul Nasir (99.43)
Vinay Kumar (99.42)	Rustam Naryaan (99.42)	Tejeswar Reddy (99.39)	Pruthvi Raj (99.39)	Debdut Saini (99.38)	G Sai Kiran (99.37)	Sahil Kumar (99.32)	Pankaj (99.31)
Satyam Agarwal (99.31)	Divyanshu Yadav (99.28)	Piyush Tiwari (99.26)	Alisha Verma (99.23)	Avinash Kumar (99.19)	Amisha Verma (99.15)	Chetan Singh (99.13)	Ritik umar (99.13)
Shubham Kumar (99.10)	Randheer (99.08)	Aryansh Tripathi (99.06)	Saurabh Maurya (99.03)	Mohammad Affan (99.03)	Pradumna Awasthi (99.02)	Harsh (99.02)	Abhay (99.01)

SCHOOL INTEGRATED PROGRAM (SIP)

Tradition of Gravity Continues,
Once Again Historical Result,
100% Students Cracked
JEE Main
(Based on Last Yr Cut off)

2020

80 Out of 80
Cracked JEE Main

We had three Batches of 55, 15 and 10.

Many Top Ranks are from these Batches

2019

79 Out of 80 in JEE Main	50 Out of 79 in JEE Adv.

2018

83 Out of 85 in JEE Main	62 Out of 83 in JEE Adv.

2017

80 Out of 85 in JEE Main	63 Out of 80 in JEE Adv.

2016

39 Out of 40 in JEE Main	31 Out of 39 in JEE Adv.

Selections Engineering 2019

gravity
Orienting Intelligence

Tarun

194
AIR
(General)

Aniket Agarwal

337
AIR
(General)

Shubh Sahu

494
AIR
(General)

Shlok Nemani

497
AIR
(General)

50 out of 79 Cracked JEE Advanced from SIP (School Integrated Program)

4 Ranks under 500 (General Category) | 2 Ranks under 10 (Reserved Category)

126 Selections in JEE Advanced | 61 Students above 99 Percentile in JEE Main 2019

Sanjana

AIR - 3*

Akash

AIR - 4*

Priyanka

AIR - 68*

Bibek Lakra

AIR - 150*

Neha Raj

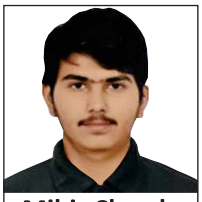
AIR - 177*

Arindam

AIR - 809
(General EWS)

Priyam

AIR - 1378
(General)



Mihir Chawla

AIR - 2237
(General)

Madhur Kumar

AIR - 2382
(General)

Manish Kumar

AIR - 2388
(General)

Saumya Raj

AIR - 2656
(General)

Raghav

AIR - 2659
(General)

Ritveek

AIR - 2709
(General)

Vanshaj

AIR - 2787
(General)

Subir Gupta

AIR - 2881
(General)

Aryan Rastogi

AIR - 3167
(General)

Devansh

AIR - 3600
(General)

Abhisht Bose

AIR - 3784
(General)